

1997 Maritime Mathematics Competition

1. How many 1 cent coins (pennies), 5 cent coins (nickels), 10 cent coins (dimes) and 25 cent coins (quarters) would be worth 93 cents if there were eight coins altogether? Is there more than one solution?
2. Find the value of xyz given that:

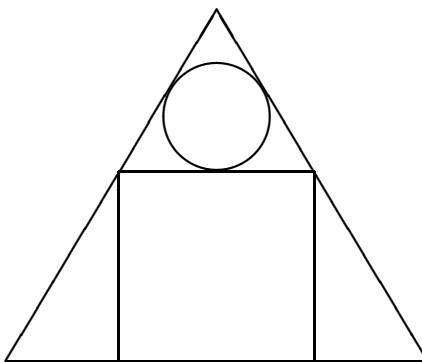
$$\begin{aligned}x + y + z &= 1 \\x^2 + y^2 + z^2 &= 2 \\x^3 + y^3 + z^3 &= 3.\end{aligned}$$

3. Show that for all natural numbers n :

$$n! \leq \left(\frac{n+1}{2}\right)^n.$$

Note that $n! = (1)(2)(3) \cdots (n-1)(n)$.

4. A circle and a square are inscribed inside an equilateral triangle as shown. If the sides of the triangle have unit length, what is the radius of the circle?



5. Let $p(x) = 1 + a_1x + a_2x^2 + \cdots + a_nx^n$, where a_1, a_2, \dots, a_n are integers such that $a_1 + a_2 + \cdots + a_n$ is an even number. Show that there are no integer solutions to the equation $p(x) = 0$.
6. A (circular) coin of radius r is dropped onto a floor which is tiled with equilateral triangles with sides of length l . What is the probability that the coin comes to rest lying completely atop one tile with no overlap onto another tile?