1998 Maritime Mathematics Competition

1. Consider all possible numbers between 100 and 1000 which are formed by using only the digits 0, 1, 2, 3, 5 (with no digit repeated). How many of these are divisible by 6?

2. A circle of radius 5 is circumscribed by a right-angled isosceles triangle. What is the length of the hypotenuse of the triangle?

3. Two trains are travelling on parallel tracks. One train is \( x \) times as fast as the other train. It takes \( x \) times as long for the two trains to pass when going in the same direction as it takes the two trains to pass when going in opposite directions. Find \( x \).

4. Show that

\[ \frac{(3\sqrt{3} + 5)^{1/3} + (3\sqrt{3} - 5)^{1/3}}{2^{2/3}\sqrt{3}} = 1. \]

5. The numbers

\[ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{99}, \frac{1}{100} \]

are written on a blackboard. Two numbers \( a \) and \( b \) are selected arbitrarily from the list, deleted, and replaced by the single number \( a + b + ab \). This is done repeatedly until one number is left. What are the possible values of this number?

6. There are \( n^k \) possible lists \((a_1, a_2, \ldots, a_k)\) which can be constructed by choosing \( k \) numbers \( a_i \) from the set \( \{1, 2, \ldots, n\} \). (Repetitions are allowed.) For each of these lists, the smallest number is noted. Prove that the sum of all these smallest numbers is

\[ 1^k + 2^k + \cdots + n^k. \]