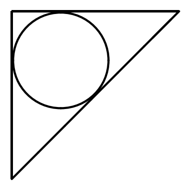
1998 Maritime Mathematics Competition

1. Consider all possible numbers between 100 and 1000 which are formed by using only the digits 0, 1, 2, 3, 5 (with no digit repeated). How many of these are divisible by 6?

2. A circle of radius 5 is circumscribed by a right-angled isoceles triangle. What is the length of the hypotenuse of the triangle?



3. Two trains are travelling on parallel tracks. One train is x times as fast as the other train. It takes x times as long for the two trains to pass when going in the same direction as it takes the two trains to pass when going in opposite directions. Find x.

4. Show that
$$\frac{(3\sqrt{3}+5)^{1/3}+(3\sqrt{3}-5)^{1/3}}{2^{2/3}\sqrt{3}}=1.$$

5. The numbers
$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{99}, \frac{1}{100}$$
are written on a blackboard. Two numbers a and b are selected arbitrarily from the list, deleted, and replaced by the single number $a+b+ab$. This is done repeatedly until one number is left. What are the possible values of this number?

6. There are n^k possible lists $(a_1, a_2, \dots, a_k)$ which can be constructed by choosing k numbers a_i from the set $\{1, 2, \dots, n\}$. (Repetitions are allowed.) For each of these lists, the smallest number is noted. Prove that the sum of all these smallest numbers is
$$1^k + 2^k + \cdots + n^k.$$