

Name _____

BARCLAY

KUCEROVSKY

SANKEY

TINGLEY

Instructions

100 points

Answer all questions on the paper provided. Show your work! All answers must be justified by showing work. Full credit will not be given for answers alone.

Use of calculators, notes, books, or scratch paper is not permitted.

1. [10 points]

(a) Find the cosine of the angle between $\mathbf{u} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$.

(b) Find the projection of \mathbf{u} onto \mathbf{v} .

2. [12 points] Consider a plane containing the vectors $\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$ and the point $(2, 3, 5)$.

(a) Find a normal vector for the plane.

(b) Find the equation of the plane.

(c) Find the equation of the line through $(3, 2, 1)$ that is perpendicular to the plane.

3. [11 points]

(a) Solve the system of equations below using any method learned in this course.

$$-2x + y + z = -9$$

$$x + y + z = 0$$

$$x + 3y = 3$$

(b) Each of the equations above represents a plane in \mathbf{R}^3 . Describe the intersection of the three planes.

4. [10 points] Find A^{-1} if $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$.

5. [10 points] Find the indicated determinant.

(a) $\begin{vmatrix} 2 & -2 & 1 \\ 4 & 0 & 3 \\ 3 & 6 & 0 \end{vmatrix}$

(b) $|LU|$, where $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 9 & 1 & 0 & 0 \\ 7 & 12 & 1 & 0 \\ 4 & -2 & 3 & 1 \end{bmatrix}$ and $U = \begin{bmatrix} 3 & 5 & 0 & 6 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & -2 & 7 \\ 0 & 0 & 0 & 4 \end{bmatrix}$.

6. [12 points]

(a) Find a basis for the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ -2 \\ 4 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 4 \\ 4 \\ -4 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} -2 \\ 5 \\ 3 \\ -7 \end{bmatrix} \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ 8 \\ 9 \\ -7 \end{bmatrix}$$

(b) What is the dimension of the subspace spanned by \mathbf{v}_1 , \mathbf{v}_2 , \mathbf{v}_3 , and \mathbf{v}_4 ?

7. [10 points] The matrix A can be reduced to the row echelon form R , given below.

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find the rank of A .

(b) Find the dimension of the null space of A .

(c) Find a basis for the column space of A .

(d) Is the vector $[1, 2, 3, 4]$ in the row space of A ?

8. [15 points] Answer (a)–(d) with regard to the matrix $B = \begin{bmatrix} 3 & -2 & 5 \\ 1 & 0 & 5 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Show that $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector for B with eigenvalue 2.

(b) Find all eigenvalues for B .

(c) Find a basis for the eigenspace of B associated with the eigenvalue 2.

(d) Suppose that k is **not** an eigenvalue of B . Is $B - kI$ invertible?

9. [10 points]

(a) Find the polar form of the complex number $8i$.

(b) Find all (complex) solutions to the equation $z^3 = 8i$. That is, find all cube roots of $8i$.

(c) Sketch the cube roots in the complex plane.