Math 1503 Final Exam December 9, 2008

Total points: 85 Name: ____________________

Total pages: 8 **Student Number:** ____________________

Please circle your instructor's name below.

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Instructions: Calculators and other electronic devides are prohibited. Show your steps and calculations, so that your answers are justified. Please use the back of the page if you need more room for your solution(s); and indicate that you have done so.

[5] 1. (a) Find the equation of the plane containing the points $(3, -1, 4)$, $(-4, -2, 10)$, and $(4, 0, 1)$.

[5] (b) Find the vector form **or** the parametric form of the equation of the line through the point $(6, -1, 1)$ that is perpendicular to the plane in part (a).

[10] 2. Solve this system of equations using any method learned in this course. Describe the solutions, if any.

$$\begin{array}{rcrcrcrcr} x & + & y & + & z & + & w & = & 10 \\ & & y & - & z & & & = & 8 \\ & & 3y & + & 2z & - & 5w & = & -1 \end{array}$$

3. For this question, let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$.

[5] (a) Show that $\mathcal{B} := \{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ forms a basis for $\mathbf{R}^3$. Any method or test from class will suffice.

[5] (b) Let $\mathcal{E}$ be the standard basis $\{\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}\}$. Find the change-of-basis matrix from $\mathcal{E}$ to $\mathcal{B}$. That is, find $A_{\mathcal{B} \leftarrow \mathcal{E}}$.

[2] (c) For the vector $z = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$, find $[z]_{\mathcal{B}}$. That is, write z in coordinates relative to the new basis $\mathcal{B}$.

4. Find the determinant of each matrix below.

[3] (a) $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$

[3] (b) $\begin{vmatrix} 2 & 4 & 0 & 1 \\ 3 & -1 & 0 & 5 \\ -2 & 1 & 0 & 1 \\ 3 & 2 & 0 & 1 \end{vmatrix}$

[3] (c) $\begin{vmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -3 & 4 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 5 & -1 \end{vmatrix}$

[3] 5. Show that $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 2 & 4 & -6 \end{bmatrix}$ is **not** invertible.

6. Let T be the linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ given by rotation of 90 degrees counter-clockwise, followed by reflection in the x-axis.

[5] (a) Find the matrix of T with respect to the standard basis.

[2] (b) Find $T\left(\begin{bmatrix} 4 \\ 7 \end{bmatrix}\right)$.

[5] (c) Are there any vectors fixed by T? That is, are there any vectors $\mathbf{v}$ with $T\mathbf{v} = \mathbf{v}$?

7. For this question, let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

[5] (a) Find the eigenvalues of A.

[5] (b) For each eigenvalue you found in (a), find an eigenvector.

[2] (c) Is A diagonalizable? If so, find a matrix P such that $P^{-1}AP$ is diagonal.

[4] 8. (a) Find the two solutions to $z^2 + 2z + 4 = 0$ and sketch them in the complex plane.

[4] (b) Call the solutions from part (a) p and q. Compute p^3 and $p + q$.

[4] (c) Use parts (a) and (b) above, or any other method, to find all solutions to the equation $z^3 = 8$.

9. Suppose that $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ are linearly independent vectors in $\mathbf{R}^3$. Let $A = [\mathbf{u}\ \mathbf{v}\ \mathbf{w}]$ (so A is the matrix with $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ as columns). Answer **TRUE** or **FALSE** for each question below. You need not justify your answers.

[1] (a) The reduced row echelon form of A is the identity matrix.

[1] (b) The rank of A is 3.

[1] (c) The vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a linear combination of $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$.

[1] (d) The system with augmented matrix $[\ A \mid \mathbf{b}\ ]$ has a unique solution for any vector $\mathbf{b}$ in $\mathbf{R}^3$.

[1] (e) A is invertible.