University of New Brunswick **Department of Mathematics & Statistics**

Math 1503 Final Exam April 20, 2009

Total points: 80 Name: ______________________

Total pages: 2 Student Number: ______________________

Please circle your instructor's name below.

DALKIR SANKEY

Instructions: Calculators and other electronic devides are prohibited. Write your solutions in the booklet provided, **not** on this sheet. Show your steps and calculations, so that your answers are justified.

1. [10 points] Answer (a)–(j) using a short calculation or explanation.

(a) Does the plane $4x - 3y + 2z = 17$ pass through the origin?

(b) Find the determinant:

$$\begin{bmatrix} 5 & 4 & 3 \\ 0 & -1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) Convert to polar form: $-5i$.

(d) Which of the following are elementary matrices?

$$(i)\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (ii)\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad (iii)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(iv)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (v)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(e) Find a vector perpendicular to $\mathbf{v} = \langle 1, 2, 3 \rangle$.

(f) Find a vector **not** in $\mathrm{Span}(\langle 1, -1, 0 \rangle, \langle 0, 0, 1 \rangle)$.

(g) Is $\begin{bmatrix} 2 & 0 \\ 5 & 6 \end{bmatrix}$ invertible?

(h) Find the length of $\mathbf{v} = \langle 5, 0, 9 \rangle$.

(i) Find the cross product of $\mathbf{v} = \langle 9, 1, 4 \rangle$ with itself.

(j) Give an example of two matrices, A and B, such that AB is defined but BA is not.

2. [10] Solve $A\mathbf{x} = \mathbf{b}$ using row reduction or another method learned in this course.

$$A = \begin{bmatrix} 4 & 2 & 0 \\ -2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 4 \\ 12 \\ -1 \end{bmatrix}$$

3. [10] Find the distance between the (parallel) planes: $3x + y - z = 8$ and $3x + y - z = 12$.

4. [10] Let S be the top half of the xy-plane. That is, $S = \{\langle x, y\rangle \mid y \geq 0\}$. Determine whether S is a subspace of $\mathbb{R}^2$ and explain your reasoning.

5. [10] Answer (a) and (b) for the given set of vectors.

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

   (a) Show that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is a basis for $\mathbb{R}^4$.

   (b) Express

$$\mathbf{b} = \begin{bmatrix} 1 \\ 5 \\ 0 \\ 3 \end{bmatrix}$$

   as a linear combination of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and $\mathbf{x}$.

6. [10] Find the eigenvalues for the matrix A and give a basis for each eigenspace of A.

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 8 & -3 & 8 \\ 0 & 0 & 5 \end{bmatrix}$$

7. [10] The eigenvalues and eigenvectors for the matrix A are given, in no particular order. Your task is to diagonalize the matrix A, and use this to compute A^5.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 5 & -3 \\ -3 & 6 & -4 \end{bmatrix} \quad \text{Eigenvalues: } 1, 2, -1. \quad \text{Eigenvectors: } \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

8. [10] Find and plot all (complex) sixth roots of -1. That is, find all solutions to the equation

$$w^6 = -1$$

   and plot your solutions in the complex plane. Label the points in your plot.