

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 3243

FINAL EXAMINATION
APRIL 1999

TIME: 3 HOURS

VALUE

- (5) 1. (a) Show that $\lim_{z \rightarrow 0} \frac{z}{\bar{z}}$ does not exist.
- (7) (b) Show that $f(r, \theta) = e^{-\theta}(\cos(\ln r) + i \sin(\ln r))$ satisfies the Cauchy-Riemann conditions for $r > 0$ and find $f'(r, \theta)$.
2. Express in the form $a + bi$, $a, b \in \mathbb{R}$.
- (4) (a) $\cos\left(\frac{\pi}{3} - i\right)$.
- (6) (b) All solutions to $z^2 + 2z + 1 - i = 0$.
- (7) (c) All solutions to $e^{2z} + e^z + 1 = 0$.
- (6) 3. If $z = x + iy$, for what x and y are all values of 1^z real?
4. Let C be the straight line from $(1, 0)$ to $(0, 1)$. Evaluate
- (6) (a) $\int_C \bar{z} dz$;
- (2) (b) $\int_C z^8 dz$.
- (7) 5. Compute $\int_C \frac{dz}{z^{1/2}}$ where C is the circle $|z| = 1$ and $z^{1/2}$ refers to the principal branch.
6. Evaluate $\int_C \frac{\cosh z^2}{z(z^2 + 4)}$ where C is
- (6) (a) $|z - i| = 2$;
- (2) (b) $|z - 2| = 1$.

7. Compute

(6) (a) $\int_C \frac{2z^4 + 1}{(3z^3 - 1)(4 + z^2)} dz$ where C is $|z| = 3$.

(2) (b) $\int_C \frac{f'(z)}{f(z)} dz$ where $f(z) = \frac{(z-4)^3(z^2+1)}{2-z^2}$ and C is $|z| = 5$.

(7) 8. Find a Laurent Series in powers of z for $f(z) = \frac{(z^3+1)^2}{z^4(1+z)(2z-1)}$ and hence find the residue of f at $z = 0$.

(7) 9. Evaluate **ONE** of the following using residues.

(a) $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2} dx.$

OR

(b) $\int_0^\pi \frac{d\theta}{(3+2\cos\theta)^2}.$

(80)