

Name: \_\_\_\_\_

Id No.: \_\_\_\_\_

**Department of Mathematics & Statistics**  
**STAT 2593 Section 1A (McLean — 9:30 a.m.)**  
**Midterm Examination**  
**October 19, 1998**

**TIME:** 50 minutes. **Total marks:** 40. You are permitted to use your textbook, and one 8 & 1/2 × 11 “crib sheet” of notes and formulae in this exam.

**SHOW ALL WORK!**

1. Suppose you fly to Toronto on Great Western Airlines, and then on to Vancouver on Air Camelia. Let  $A$  be the event that GWA loses your luggage, and  $B$  be the event that AC loses your luggage. Suppose that  $P(A) = 0.2$ ,  $P(B) = 0.15$  and  $P(A \cup B) = 0.3$ .
  - (a) What is the probability that both airlines lose your luggage?
  - (b) What is the probability that exactly one of the airlines loses your luggage?
  - (c) What is the probability that both airlines lose your luggage given that at least one of them loses your luggage?
  - (d) Are the events  $A$  and  $B$  mutually exclusive? Explain briefly.
  - (e) Are the events  $A$  and  $B$  independent? Explain briefly.

2. Suppose that the (directed) distance in centimetres between a point target on a line and a shot aimed at that target is a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{32}(a^2 - x^2) & \text{for } -a \leq x \leq a \\ 0 & \text{elsewhere.} \end{cases}$$

- (a) What must the constant  $a$  be equal to?
- (b) What is the expected distance of the shot from the target? (Hint: **Don't** calculate. Think. What is the shape of the p.d.f.?)
- (c) What is the **variance** of the distance of the shot from the target?
- (d) What is the probability that the shot lands within 1/2 cm of the target?

3. (a) Suppose that from a shipment of 15 concrete cylinders that has been received by a contractor, 5 cylinders are selected (at random, of course!) for use in a contract with the Provincial Government. Suppose that of the 15, 6 have a crushing strength below the specified minimum. Calculate the probability that exactly 3 of the cylinders for the Provincial Government project have inadequate crushing strength.
- (b) Now suppose that the contractor receives a shipment of 1500 concrete cylinders, and that 600 of these have crushing strength below the specified minimum. Again 5 cylinders are selected at random for use in a contract with the Provincial Government. In this setting what is the probability that exactly 3 of the cylinders for the Provincial Government project have inadequate crushing strength.
- (c) For the second scenario (i.e. the 5 cylinders are selected from a shipment of 1500, 600 of which have inadequate crushing strength) find the probability that **at most 2** of the cylinders for the Provincial Government have inadequate crushing strength.

4. Students in a certain Statistics class attempt to guess the age of their Statistics lecturer. Suppose that their guesses are normally distributed with mean 25 years, and standard deviation 2.7 years.
- (a) What is the probability that one such guess falls between 24.5 and 26 years?
  - (b) Suppose that three students average their guesses (i.e. they take the arithmetic mean). What is the probability that this mean lies between 24.5 and 26 years?
  - (c) Suppose that the lecturer takes guesses sequentially, and says she will give a caramel to the first student whose guess falls within the interval that she has in mind. (This interval is the interval from 24.5 to 26 years, but of course she doesn't tell them this. The guesses are not revealed until one is successful, hence they may be considered to be independent random variables.) What is the probability that the first guess to fall in the interval from 24.5 to 26 will be the 4<sup>th</sup> one?