DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

Final Examination December, 2001 Time: 3 Hours

NO CALCULATORS

MARKS

1. Find the derivative of each of the following functions **(do not simplify your answers):**

(4) (a) $f(x) = (x^3 + \frac{2}{x})^3$

(4) (b) $g(t) = \ln(\cos t) - \sin(\ln t)$

(4) (c) $f(x) = \dfrac{x-1}{e^x + 1}$

(4) (d) $y = \sinh(x^2)\tan x$

(4) (e) $f(x) = \arctan(\sqrt{x})$ (i.e. $\tan^{-1}\sqrt{x}$)

(4) (f) $f(x) = \sin^{-1}(1-3x)$ (i.e. $\arcsin(1-3x)$)

(4) (g) $y = (x+1)^{(1-x)}$

(8) 2. (a) Use logarithmic differentiation to verify the well-known formula for the derivative of

$$f(x) = a^x \quad (a > 0).$$

(You cannot simply quote the known formula!)

(b) Use implicit differentiation to find the slope of the tangent line to the curve with equation

$$x^5 + xy + y^7 = 1,$$

at the point P where this curve crosses the y-axis.

(8) 3. Use the definition of the derivative as a limit to find the derivative of

$$f(x) = \sqrt{3x+1}.$$

(9) 4. Evaluate the following limits.

(a) $\lim\limits_{h\to 0} \dfrac{(h-5)^2 - 25}{h}$

(b) $\lim\limits_{x\to -\infty} \dfrac{1+3x}{\sqrt{2x^2+x}}$

(c) $\lim\limits_{x\to 0} \dfrac{e^x - 2x - 1}{\sin x}$

(8) 5. (a) Find the inverse of the function $f(x) = 3e^x + 2$. What is the domain and range of the inverse?

(b) Solve the following equation for x

$$\ln(2x+1) - \ln x = 1$$

(10) 6. A light is on the ground 40 feet from a building. A 6 foot man is walking from the light toward the building at 6 feet per second. How rapidly is the shadow on the building growing shorter when he is 30 feet from the light?

(5) 7. Find all vertical and horizontal asymptotes for

$$y = \frac{2x^2 + x + 1}{x^2 - 1}.$$

(Sketch not required.)

8. Consider the graph of the function $f(x) = x^3 + 3x^2 - 9x$

(2) (a) Find the intercepts.

(4) (b) Determine where $f(x)$ is increasing or decreasing. Find all local maxima and minima.

(4) (c) Determine where $f(x)$ is concave up or concave down. Find the inflection points.

(4) (d) Sketch the graph of $f(x)$.

(10) 9. The space within a race track of lenth 2 km is to consist of a rectangle with a semi-circular area at each end. To what dimensions should the track be built to maximize the area of the rectangle? Justify why there is a maximum.

(100)