

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

FINAL EXAMINATION
APRIL 2003

TIME: 3 HOURS
TOTAL POINTS = 100

INSTRUCTIONS:

- (a) You must show all calculations for full marks.
- (b) Calculators **are not** permitted.

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MARKS

1. Find $\frac{dy}{dx}$ (do not simplify the answers):

(3) (a) $y = \left(\frac{x^2}{8} + x - \frac{1}{\sqrt{x}}\right)^4$

(3) (b) $y = e^{3x} \ln x$

(3) (c) $y = (\sin x^2)(\cos 2x)$

(3) (d) $y = 3^x + \ln\left(\frac{\sqrt{x+1}}{x}\right)$

(3) (e) $y = \frac{1-x}{1+x^2}$

(3) (f) $y = \tan\left(\pi - \frac{1}{x}\right)$

(3) (g) $y = \sin^{-1}(3x) + \cosh 2x$

(4) 2. (a) Use logarithmic differentiation to find the derivative of $y = x^{5x}$.

(4) (b) Use implicit differentiation to find the derivative of $x^3 + 2y^3 - 9xy = 7$.

(4) (c) Calculate $f''(0)$ for the function $f(x) = e^{x^2}$.

3. Evaluate the following limits:

(3) (a) $\lim_{x \rightarrow 3} \frac{x-3}{x^2-2x-3}$

(3) (b) $\lim_{x \rightarrow \infty} \frac{1-2x+x^3}{2+3x-4x^3}$

(3) (c) $\lim_{x \rightarrow 0} \frac{e^x-1}{\sin 2x}$

(6) 4. Find the equation of the tangent line to $y = \ln 3x$ at the point where the tangent line is parallel to the line $2y - 6x = 1$.

(6) 5. Using the limit definition of the derivative, find $\frac{dy}{dx}$ if $y = \sqrt{x}$.

- (3) 6. (a) Solve for x : $\ln(2x - 5) - \ln x = 0$.
- (3) (b) Find the inverse of $y = 2e^{x+1}$. Give the domain and range of the inverse.
- (4) (c) Evaluate:
- (i) $\cos^{-1}\left(\sin \frac{4\pi}{3}\right)$.
- (ii) $\cos\left(\sin^{-1} \frac{3}{5}\right)$.
- (8) 7. A 5 foot ladder that was resting against a wall starts sliding. The top remains in contact with the wall, and the bottom moves away from the wall at a rate of 2 ft/sec. How fast is the top of the ladder moving downwards when the bottom is 4 feet from the wall.
- (6) 8. A rectangle is inscribed in an isosceles triangle as shown in the diagram below. The height of the triangle is 20 cm. and its base is 12 cm. Find the dimensions of the rectangle which has maximum area.
- (4) 9. Determine the horizontal and vertical asymptotes of the graph of $f(x) = \frac{3 - 2x^2}{x^2 + 2x}$.
- (18) 10. Consider the graph of the function $f(x) = x^4 - 2x^2$.
- (a) Find the x and y intercepts of $f(x)$.
- (b) Find the intervals where $f(x)$ is increasing, and find the intervals where $f(x)$ is decreasing.
- (c) Find all the local maxima and minima points.
- (d) Find the intervals where $f(x)$ is concave up, and the intervals where $f(x)$ is concave down.
- (e) Give the points of inflection.
- (f) Sketch the graph of $f(x)$.