

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

FINAL EXAMINATION
DECEMBER 2004

TIME: 3 HOURS
TOTAL POINTS = 100

MARKS

1. Find the derivatives of the following functions. (**Do NOT simplify answers!**)

(4) (a) $f(x) = \frac{1 + \sin 2x}{5 + \cos 3x}$

(4) (b) $f(x) = x^{5/3} \sqrt{3 + x^2}$

(4) (c) $f(x) = (\sin^{-1} x)(\sinh x)$ (In other notation, $f(x) = (\arcsin x)(\sinh x)$)

(4) (d) $f(x) = \ln(x + x^2) + \tan^2 x$

(4) (e) $f(x) = x^x$

2. Differentiate the following. **Simplify** your answers.

(4) (a) $y = \ln(e^x \cdot \cos x)$

(4) (b) $y = \sin^{-1}(\cos x)$ (In other notation, $y = \arcsin(\cos x)$)

(4) (c) $y = \frac{x + 1}{(x^2 - 1)^{100}}$

(4) 3. Use implicit differentiation to show that $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$. (Recall: $y = \tan^{-1} x$ means that $\tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$).

4. Consider the curve $x^2 + 2xy + 10y^2 = 9$.

(4) (a) Find $\frac{dy}{dx}$.

(1) (b) Verify that $(3, 0)$ is a point on the curve.

(2) (c) Find the equation of the line tangent to the curve at the point $(3, 0)$.

(3) (d) Find both points on the curve where the tangent line is horizontal.

- (4) 5. Find a formula for the inverse function of $f(x) = \frac{1}{2} \ln(x - 1)$.
- (3) 6. (a) Find $f(x)$ if $f'(x) = e^x - \sin(2x)$
- (3) (b) Find $f(x)$ if $f'(x) = x + \sqrt{x}$ and $f(1) = 3$
- (3) 7. Find the area between the graph of $y = x^3 + x + 2$ and the x -axis, $-1 \leq x \leq 2$.
- (3) 8. An object moves with velocity (in meters/second) given by

$$v = \frac{1}{t}.$$

How far does the object move between time $t = 1$ second and $t = 10$ seconds?

9. Find the exact values of the following:
- (2) (a) $\tan\left(\sin^{-1} \frac{4}{5}\right)$
- (2) (b) $e^{2 \ln 5}$
10. Find all horizontal and vertical asymptotes for
- (3) (a) $y = \frac{x^2 + 3x - 1}{x^2 - 1}$
- (2) (b) $y = \ln x$
11. Consider the function $f(x) = 2x^3 - 3x^2$.
- (4) (a) Determine the intervals where $f(x)$ is increasing, where $f(x)$ is decreasing, and all local maxima and minima of $f(x)$, if any.
- (4) (b) Find the intervals on which $f(x)$ is concave up, concave down, and the points of inflection, if any.
- (4) (c) Sketch the graph of $y = f(x)$. Be sure that your graph indicates the information you have found in (a) - (b).

12. Evaluate the following limits:

(3) (a) $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 3x}$

(3) (b) $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7}$

(3) (c) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

- (8) 13. Fred wishes to make himself a BIG birthday cake. He knows that he and his friends will eat a total of $\frac{1}{16}$ cubic meter of cake. The cake is to have a square base and 4 vertical sides. Fred doesn't like icing so he wishes to minimize the combined area of the top and sides. What should the dimensions of the cake be?

