

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

FINAL EXAMINATION  
DECEMBER 2004

TIME: 3 HOURS  
TOTAL POINTS = 100

MARKS

1. Find the derivatives of the following functions. (**Do NOT simplify answers!**)

(4) (a)  $f(x) = \frac{1 + \sin 2x}{5 + \cos 3x}$

(4) (b)  $f(x) = x^{5/3} \sqrt{3 + x^2}$

(4) (c)  $f(x) = (\sin^{-1} x)(\sinh x)$  (In other notation,  $f(x) = (\arcsin x)(\sinh x)$ )

(4) (d)  $f(x) = \ln(x + x^2) + \tan^2 x$

(4) (e)  $f(x) = x^x$

2. Differentiate the following. **Simplify** your answers.

(4) (a)  $y = \ln(e^x \cdot \cos x)$

(4) (b)  $y = \sin^{-1}(\cos x)$  (In other notation,  $y = \arcsin(\cos x)$ )

(4) (c)  $y = \frac{x + 1}{(x^2 - 1)^{100}}$

(4) 3. Use implicit differentiation to show that  $\frac{d}{dx} \tan^{-1} x = \frac{1}{1 + x^2}$ . (Recall:  $y = \tan^{-1} x$  means that  $\tan y = x$  and  $-\frac{\pi}{2} < y < \frac{\pi}{2}$ ).

4. Consider the curve  $x^2 + 2xy + 10y^2 = 9$ .

(4) (a) Find  $\frac{dy}{dx}$ .

(1) (b) Verify that  $(3, 0)$  is a point on the curve.

(2) (c) Find the equation of the line tangent to the curve at the point  $(3, 0)$ .

(3) (d) Find both points on the curve where the tangent line is horizontal.

- (4) 5. Find a formula for the inverse function of  $f(x) = \frac{1}{2} \ln(x - 1)$ .
- (3) 6. (a) Find  $f(x)$  if  $f'(x) = e^x - \sin(2x)$
- (3) (b) Find  $f(x)$  if  $f'(x) = x + \sqrt{x}$  and  $f(1) = 3$
- (3) 7. Find the area between the graph of  $y = x^3 + x + 2$  and the  $x$ -axis,  $-1 \leq x \leq 2$ .
- (3) 8. An object moves with velocity (in meters/second) given by

$$v = \frac{1}{t}.$$

How far does the object move between time  $t = 1$  second and  $t = 10$  seconds?

9. Find the exact values of the following:
- (2) (a)  $\tan\left(\sin^{-1} \frac{4}{5}\right)$
- (2) (b)  $e^{2 \ln 5}$
10. Find all horizontal and vertical asymptotes for
- (3) (a)  $y = \frac{x^2 + 3x - 1}{x^2 - 1}$
- (2) (b)  $y = \ln x$
11. Consider the function  $f(x) = 2x^3 - 3x^2$ .
- (4) (a) Determine the intervals where  $f(x)$  is increasing, where  $f(x)$  is decreasing, and all local maxima and minima of  $f(x)$ , if any.
- (4) (b) Find the intervals on which  $f(x)$  is concave up, concave down, and the points of inflection, if any.
- (4) (c) Sketch the graph of  $y = f(x)$ . Be sure that your graph indicates the information you have found in (a) - (b).

12. Evaluate the following limits:

(3) (a)  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 3x}$

(3) (b)  $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x-7}$

(3) (c)  $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

- (8) 13. Fred wishes to make himself a BIG birthday cake. He knows that he and his friends will eat a total of  $\frac{1}{16}$  cubic meter of cake. The cake is to have a square base and 4 vertical sides. Fred doesn't like icing so he wishes to minimize the combined area of the top and sides. What should the dimensions of the cake be?

