

# DEPARTMENT OF MATHEMATICS & STATISTICS

## MATH 1003

FINAL EXAMINATION  
DECEMBER 2005

TIME: 3 HOURS  
TOTAL POINTS = 100

### MARKS

1. Find the derivatives of the following functions. (Do NOT simplify answers!)

(4) (a)  $f(x) = \sqrt{3 + x^2} \tan x$

(4) (b)  $f(x) = \frac{e^x}{1 + \cos(2x)}$

(4) (c)  $f(x) = e^{5x} \ln(7x)$

(4) (d)  $f(x) = (\cosh x)^2 + \sinh(x^2)$

(4) (e)  $f(x) = (\sin x)^x$

2. Differentiate the following. Simplify your answers.

(4) (a)  $f(x) = \tan^{-1}(\sqrt{x^2 + 1})$

(4) (b)  $f(x) = \ln(\sec x + \tan x)$

(4) (c)  $f(x) = \frac{x - 1}{(x + 2)^{50}}$

3. Consider the curve  $x^3 + y^3 = 9$ .

(4) (a) Find  $\frac{dy}{dx}$ .

(1) (b) Verify that (1, 2) is a point on the curve.

(2) (c) Find the equation of the line tangent to the curve at the point (1, 2).

4. Evaluate the following limits:

(3) (a)  $\lim_{x \rightarrow \infty} \frac{10x^2}{e^x}$

(3) (b)  $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{\ln x}$

(3) (c)  $\lim_{x \rightarrow 0} \frac{x^3 - 1}{\cos(x)}$

(4) 5. (a) Find a formula for the inverse function of  $f(x) = \arcsin(e^x - 1)$ .  
(Note that  $f(x) = \sin^{-1}(e^x - 1)$  in other notation.)

(b) Find the domain of  $f(x)$  in part (a).

(3) 6. (a) Find  $\int \left( x^{1/2} + \frac{1}{x} \right) dx$

(4) (b) Find  $f(x)$  if  $f'(x) = 2x + 3x^2 + 1$  and  $f(0) = 4$ .

(4) (c) Find the area below one arch of the graph  $y = \cos x$  but above the  $x$ -axis.

7. Find all horizontal and vertical asymptotes, and give a simple sketch, for the following functions. (There is no need for details concerning increase/decrease, concavity, etc.)

(4) (a)  $y = \frac{x}{x-2}$

(3) (b)  $y = e^x$

8. Consider the function  $f(x) = x^4 - 4x^3$ .

(4) (a) Determine the intervals where  $f(x)$  is increasing, where  $f(x)$  is decreasing, and all local maxima and minima of  $f(x)$ , if any.

(4) (b) Find the intervals on which  $f(x)$  is concave up, concave down, and the points of inflection, if any.

(4) (c) Sketch the graph of  $y = f(x)$ . Be sure that your graph indicates the information you have found in (a) - (b).

(10) 9. An open-topped cylindrical pot is to be constructed with volume  $250\pi$  cm<sup>3</sup>. Material for the bottom costs 4 cents/cm<sup>2</sup>, while material for the curved sides costs 2 cents/cm<sup>2</sup>. Find the dimensions of the pot that will minimize the total cost.

Recall: Volume equals base area times height. The curved side area of the pot equals the base circumference times the height.

(4) 10. (a) Show, by giving and checking explicit values, that usually

$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

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(b) Give an example of a function  $g$  with domain  $\mathbb{R}$  such that

$$g(a+b) = g(a) + g(b)$$

for all real numbers  $a, b$ .

(c) Find the exact value of

$$\ln\left(\frac{\sqrt{e}}{e}\right)$$

.

(d) Find the exact value of

$$\cos\left(\arctan\left(\frac{1}{\sqrt{3}}\right)\right)$$

.

(4) 11. Find the equation of the line which is tangent to the curve  $y = \ln x$  and is also parallel to the line  $y = 2 + x$ .