

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

FINAL EXAMINATION
APRIL 2005

TIME: 3 HOURS
TOTAL POINTS = 100

INSTRUCTIONS:

- (a) You must show all calculations for full marks.
- (b) Calculators **are not** permitted.

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MARKS

1. Differentiate each of the following functions. **Do not simplify.**

- (3) (a) $y = (5x - 1)^3 + 2\pi$
- (3) (b) $y = \frac{x^3 - 2\sqrt{x}}{3x}$
- (3) (c) $y = \frac{\cos x}{1 - \sin x}$
- (3) (d) $y = x^2 \sinh x$
- (3) (e) $y = e^{(\cos t + \ln t)}$
- (3) (f) $y = 2^{\sin 3x}$
- (3) (g) $y = \tan^{-1} 3x$ (In other notation: $y = \arctan(3x)$)
- (3) (h) $y = x^{6x}$

2. For $x^3 + y^3 = 6xy$

- (3) (a) find y' ;
- (3) (b) find the equation of the tangent line at $(3, 3)$.

(4) 3. Use Logarithmic differentiation to find the derivative of

$$y = \frac{\sqrt{x}(x^3 + 2)^4}{(2x + 1)^5}.$$

(4) 4. Use the limit definition of the derivative to find $f'(x)$ for:

$$f(x) = \sqrt{x - 1}.$$

5. Integrate the following:

(3) (a) $\int (3e^x + \sqrt{x} + \sin x) dx$

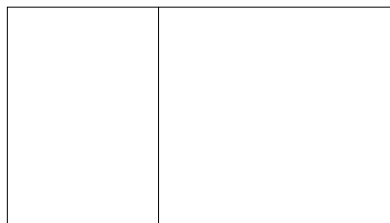
(3) (b) $\int_1^e \frac{1}{x} dx$

(3) 6. Given $f'(x) = 6x$ and $f(1) = 4$, find $f(x)$.

(3) 7. Find the area between the graph of $y = x^3 + 2x^2$ and the x -axis, from $x = 0$ to $x = 2$.

(3) 8. A ball is thrown into the air. Its height, in metres, after t seconds is given by $H(t) = 8t - 2t^2$ metres. Determine the velocity of the ball when it strikes the ground.

(7) 9. A rectangular garden area is to be fenced, as shown.



Given that there are 20 metres of fencing available, determine the maximum area of the enclosed garden.

10. Evaluate the following limits (if they exist):

(3) (a) $\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x + 2}$

(3) (b) $\lim_{x \rightarrow 1} \frac{1}{(x - 1)^2}$

(3) (c) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

(3) 11. Find the inverse of $y = e^x + 2$. State the domain and range of $y = e^x + 2$ and its inverse.

(1) 12. Give an example of a function with domain $(-\infty, 3) \cup (3, \infty)$.

(1) 13. (a) Evaluate: $\arctan(1)$.

(2) (b) Solve for x : $\sin(2x) = 0$, where $0 \leq x \leq 2\pi$.

(1) (c) Evaluate: $e^{3 \ln 2}$.

(1) (d) A table of values for $f(x)$ and $g(x)$ is provided for $x = 1, 2, 3, 4, 5$ as shown. Find the value of $f(g(2))$.

x	$f(x)$	$g(x)$
1	2	3
2	3	4
3	4	5
4	5	6
5	6	7

14. Given $f(x) = \begin{cases} 4 & \text{if } x \leq -2 \\ x^2 & \text{if } -2 < x < 1 \\ 3 - k & \text{if } x \geq 1 \end{cases}$. Find each of the following:

(1) (a) $\lim_{x \rightarrow -2^-} f(x)$

(1) (b) $\lim_{x \rightarrow -2^+} f(x)$

(1) (c) $f(-2)$

(1) (d) State the value of k for which $f(x)$ is continuous at $x = 1$.

(3) 15. Find the horizontal and vertical asymptotes of

$$y = \frac{x^2}{x^2 - 3x - 10}.$$

(15) 16. For the function $f(x) = 2x^3 - 9x^2 - 24x - 12$, determine:

(a) intervals of increase and decrease for $f(x)$;

(b) intervals of concavity for $f(x)$;

(c) all local maxima and minima points;

(d) points of inflection.

(e) Sketch the function.