

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

FINAL EXAMINATION  
DECEMBER 2006

TIME: 3 HOURS  
TOTAL POINTS = 90

INSTRUCTIONS:

- (a) This exam has 7 pages.
- (b) You must show all calculations for full marks.
- (c) Calculators **are not** permitted.
- (d) Do each question in the indicated space. If you need more space, use the reverse side, but clearly indicate to us where your answer is located.

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MARKS

1. Find the derivative of each function. (You may use all the rules for differentiating functions.) DO NOT SIMPLIFY YOUR ANSWER!

(2) (a)  $f(x) = 3x^4 - \frac{2}{x^{1/3}}$

(2) (b)  $f(x) = (x + 1) \sin x$

(2) (c)  $f(x) = \tan(x^2 + 4)$

(2) (d)  $f(x) = \frac{e^{-x}}{e^x + 1}$

(2) (e)  $f(x) = e^{2x}$

(3) (f)  $f(x) = \ln(\ln(1 + x))$

(2) (g)  $f(x) = \arcsin\left(\frac{x}{3}\right)$

(4) 2. (a) Find  $\frac{dy}{dx}$  if  $y^3 + xy = 2$ .

(4) (b) Find the equation of the tangent line to the curve  $y^3 + xy = 2$  at the point (1, 1).

(5) 3. Use the definition of the derivative to compute  $f'(x)$  if  $f(x) = \sqrt{1 + x}$ .

4.  $f(x) = \frac{x^2}{x^2 + 9}$ .

Below use the facts that  $f'(x) = \frac{18x}{(x^2 + 9)^2}$  and  $f''(x) = \frac{-54(x^2 - 3)}{(x^2 + 9)^3}$ .

- (3) (a) Find all critical numbers (that is, critical points and singular points) of  $f(x)$  above.

- (4) (b) Determine the intervals where  $f(x)$  is increasing, and where it is decreasing.
- (3) (c) Locate all local extreme values of  $f(x)$ .
- (4) (d) Determine the intervals where  $f(x)$  is concave up and where it is concave down.
- (3) (e) Locate all inflection points of  $f(x)$ .
- (4) (f) Find all vertical and horizontal asymptotes (if any exist) for the graph  $y = f(x)$ .
- (5) (g) Sketch the graph  $y = f(x)$  labelling the axes and show the information found in (c), (e), (f).

5. Find each limit below:

(4) (a)  $\lim_{x \rightarrow 1^-} \frac{3}{\ln(1-x)}$

(4) (b)  $\lim_{x \rightarrow \infty} \frac{e^{-x}}{\sin\left(\frac{1}{x^2}\right)}$

(4) 6. (a) Find the most general anti-derivative of  $f(x) = \frac{3}{x^2} - 2e^x$ .

(4) (b) Find the function which satisfies  $h''(t) = \frac{-1}{t^2}$  subject to  $h(1) = 0$  and  $h'(1) = 2$  and with domain  $t > 0$ .

(5) 7. Find the point on the curve  $y = x^2 + 1$  closest to the point  $(0, 3)$ .

(4) 8. (a) Show that  $f(x) = \frac{1}{2}(e^x - e^{-x})$  is an everywhere increasing function.

(4) (b) From part (a), justify that  $f(x)$  has an inverse, and find the inverse function.

9. The function  $f(t)$  is defined by

$$f(t) = \begin{cases} 2be^t & \text{for } t < 0 \\ 1 - b \cos t & \text{for } t \geq 0 \end{cases}$$

where  $b$  is a constant.

(4) (a) Compute  $\lim_{t \rightarrow 0^+} f(t)$  and  $\lim_{t \rightarrow 0^-} f(t)$ .

(3) (b) What is the value of the constant  $b$  so that  $f(t)$  is continuous at  $t = 0$ ?