STUDENT'S NAME:____________________________________ **ID #:**_________

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DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1003

FINAL EXAMINATION
DECEMBER 2009

TIME: 3 HOURS
TOTAL POINTS = 90 XXXXXXXXXX

INSTRUCTIONS:

1. This exam has 8 pages (including the cover page).
2. You must show all calculations for full marks.
3. Do each question in the indicated space. If you need more space, use the reverse side, but clearly indicate to us where your answer is located.
4. The only items allowed on your desk are writing instruments. We will supply any needed scrap paper. Calculators **are not** permitted. Neither are cell phones nor any other electronic gadget!

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XXXXXXXXXXXXXXXXXXX Perhaps better to use Pages rather than questions XXXXXXXXX

QUESTION	MARK
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MARKS

1. Find the derivatives of the following functions. (**Do NOT simplify answers!)**

(4) (a) $f(x) = \tan(5x) + \sin(x^2)$

(4) (b) $f(t) = 5(t + e^t)^7$

(4) (c) $f(x) = \dfrac{\ln(x)}{\sqrt{3 + x^2}}$

(4) (d) $f(x) = (1 + e^x)\sin^{-1}(x)$ ($\sin^{-1} x$ is the same as $\arcsin x$.)

(4) (e) $f(x) = \left(\cos\left(x + \sqrt{x + 2}\right)\right)^5$

(4) 2. Use logarithmic differentiation to find $\frac{dy}{dx}$ where $y = \sqrt{\frac{(x+5)^{11}}{(3x+1)^7}}$.

3. Differentiate the following. **Simplify** your answers.

(4) (a) $y = \frac{x-1}{(x^2-1)^5}$

(4) (b) $y = \frac{e^x \sin(x) + e^x \cos(x)}{2}$

(4) (c) $y = \ln(\sec(x) + \tan(x))$

(2) 4. (a) State the definition of the derivative.

(3) (b) Use this definition to show that the derivative of $y = \dfrac{1}{2x+1}$ is $\dfrac{dy}{dx} = \dfrac{-2}{(2x+1)^2}$.

(3) (c) What is an equation for the line tangent to $y = \dfrac{1}{2x+1}$ the point (-1,-1)?

5. Let $f(x) = \ln(x+1)$.

(2) (a) What is the domain of f?

(2) (b) What are the range of f and the domain of f^{-1}?

(2) (c) Find a formula for f^{-1}.

6. Evaluate the following limits:

(3) (a) $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

(3) (b) $\lim_{x\to 1} \dfrac{\ln x - 1}{\ln x + 1}$

7. The function $f(t)$ is defined by

$$f(t) = \begin{cases} 3t + b & \text{for} \quad t < 1 \\ 2 - bt^2 & \text{for} \quad t \geq 1 \end{cases}$$

where b is a constant.

(4) (a) Compute $\lim_{t\to 1^+} f(t)$ and $\lim_{t\to 1^-} f(t)$ in terms of b.

(3) (b) What is the value of the constant b so that $f(t)$ is continuous at $t = 1$?

(4) 8. Find, and justify with limits, all horizontal and vertical asymptotes for the following equation. Then make a simple sketch of the equation. There is <u>no</u> need for details concerning increase/decrease, concavity, etc.

$$y = \frac{x+1}{x^2-4}$$

(3) 9. Find $\int (\sqrt{x} - \cos x)dx$.
(That is, find the most general anti-derivative of $f(x) = \sqrt{x} - \cos x$.)

(3) 10. Solve the initial value problem $\dfrac{dy}{dx} = x^3 + e^x$, given $y(0) = 5$.

11. Consider the function $f(x) = -2x^3 + 6x^2 - 3$.

(Use the back of the previous page for rough work. Please put the answers to the questions below on this page.)

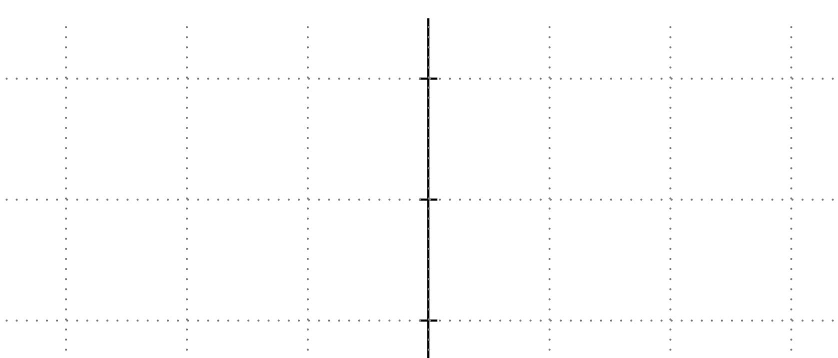
(2) (a) Determine the intervals where $f(x)$ is increasing; also determine where $f(x)$ is decreasing.

(2) (b) Find all local maxima and minima (if any) for $f(x)$.

(c) Find the absolute maximum and also the absolute minimum value of $f(x)$ on the
(2) interval $[-2, 1]$.

(3) (d) Find the intervals where $f(x)$ is concave up, where $f(x)$ is concave down and any points of inflection.

(4) (e) Sketch the graph of $f(x)$. Be sure your sketch incorporates the information from parts (a), (b), (d). Put appropriate scales on the axes.



(6) 12. A spherical balloon is being inflated at a rate of 5 cubic meters/minute. How fast is the radius of the balloon increasing at the instant the radius is 2 meters? (The volume V of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$).

(8) 13. Santa Claus has a piece of land on which he would like to grow apples. Research has shown that if he plants 24 trees, each tree will produce 600 apples per year. For each additional tree planted, the number of apples on each tree will decrease by 12 apples per year. How many trees should Santa plant to maximize his apple production?

Be sure to justify your work.