

**Math 1003 Final Examination, Fall 2010**Department of Mathematics and Statistics  
University of New Brunswick

Please circle the name of your section/instructor.

1A-Dalkir

3A-Watmough

5A-Sanke

7A-Nanayakkara

2A-Dalkir

4A-Watmough

6A-Sanke

Answer each question in the space provided. Use the reverse side for rough work. The marks for each question are shown in the left margin.

**The exam is closed book, the use of calculators is NOT permitted.**

Total Marks: 100

Time: 175 min

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1. Use the rules of differentiation to compute the derivatives of the following functions (do not simplify the answers):

(4) (a)  $y = x^6 - \frac{3}{\sqrt{x}} + \frac{5}{x^2} - 2$

(4) (b)  $y = \frac{e^{2x}}{1 - e^{x^2}}$

(4) (c)  $g(t) = \ln(\cos t) - \sin(\ln t)$

(4) (d)  $y = \arcsin\left(\frac{2}{x}\right)$  ( i.e.  $y = \sin^{-1}\left(\frac{2}{x}\right)$  )

(4) (e)  $y = (x + 1)^{(1-x)}$

2. Use the rules of differentiation to compute the derivatives of the following functions. Please simplify your answers.

(3) (a)  $f(x) = x \tan^{-1}\left(\frac{x}{2}\right) - \ln(x^2 + 4)$

(3) (b)  $g(\theta) = \ln \sec^2 \theta$

(3) (c)  $p(x) = \sqrt{\frac{x^2 + x}{x^2}}$

3. Compute these limits. Show your work. In particular, if a limit does not exist, briefly explain why.

(2) (a)  $\lim_{\theta \rightarrow \pi/2} \frac{1 - \sin \theta}{1 + \cos 2\theta}$

(2) (b)  $\lim_{x \rightarrow 0} \frac{3^x - 1}{2^x - 1}$

(2) (c)  $\lim_{t \rightarrow 0} \frac{t - 3}{t^2 - 3}$

- (8) 4. Find an equation for the line tangent to the curve defined implicitly by the relation

$$6x^2 + 3xy + 2y^2 + 17y = 6$$

at the point  $(-1, 0)$ .

- (2) 5. Evaluate the expression  $\tan(\cos^{-1} x)$ .

6. Suppose  $f$  is continuous on  $[1, 5]$  and differentiable on  $(1, 5)$ , with  $f(1) = 8$ ,  $f(5) = 4$  and  $f'(3) = 1$ . Answer each of the following, and justify your answers.

- (3) (a) Does  $f'(x) = -1$  for some  $x$  in  $(1, 5)$ ?

- (3) (b) Does  $f'(x) = 0$  for some  $x$  in  $(1, 5)$ ?

- (3) (c) Could  $f(x) > 8$  for some  $x$  in  $[1, 5]$ ?

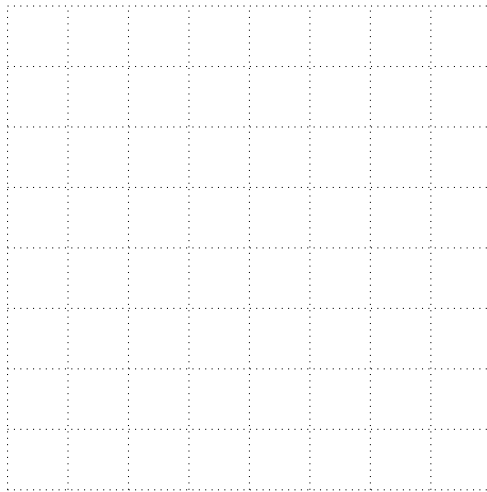
- (3) (d) Does  $f$  have an absolute minimum on  $[1, 5]$ ?

- (8) 7. Sand falls from a conveyor belt at the rate of  $10 \text{ m}^3/\text{min}$  onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast is the radius of the base of the pile increasing when the pile is 4 m high?. Note that the volume of a cone is  $\frac{1}{3}\pi r^2 h$ .

- (2) 8. (a) Find and justify with limits all vertical or horizontal asymptotes (if any) for the graph of

$$y = \frac{x - 3x^2}{x^2 - 4}.$$

- (3) (b) On the set of axes indicate the asymptotes and sketch a possible graph for the curve which indicates how the curve behaves near the asymptotes. Do not compute the derivatives of  $y$ .



- (3) 9. Find a function  $f$  satisfying  $f'(x) = \sin(\pi x/4) + \frac{1}{x}$  and  $f(2) = 0$ .

- (4) 10. Find the area under the graph of  $y = x^2 + 7$  and over the interval  $1 \leq x \leq 2$ .

11. Throughout this question,  $f$  is the function defined by  $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$ .

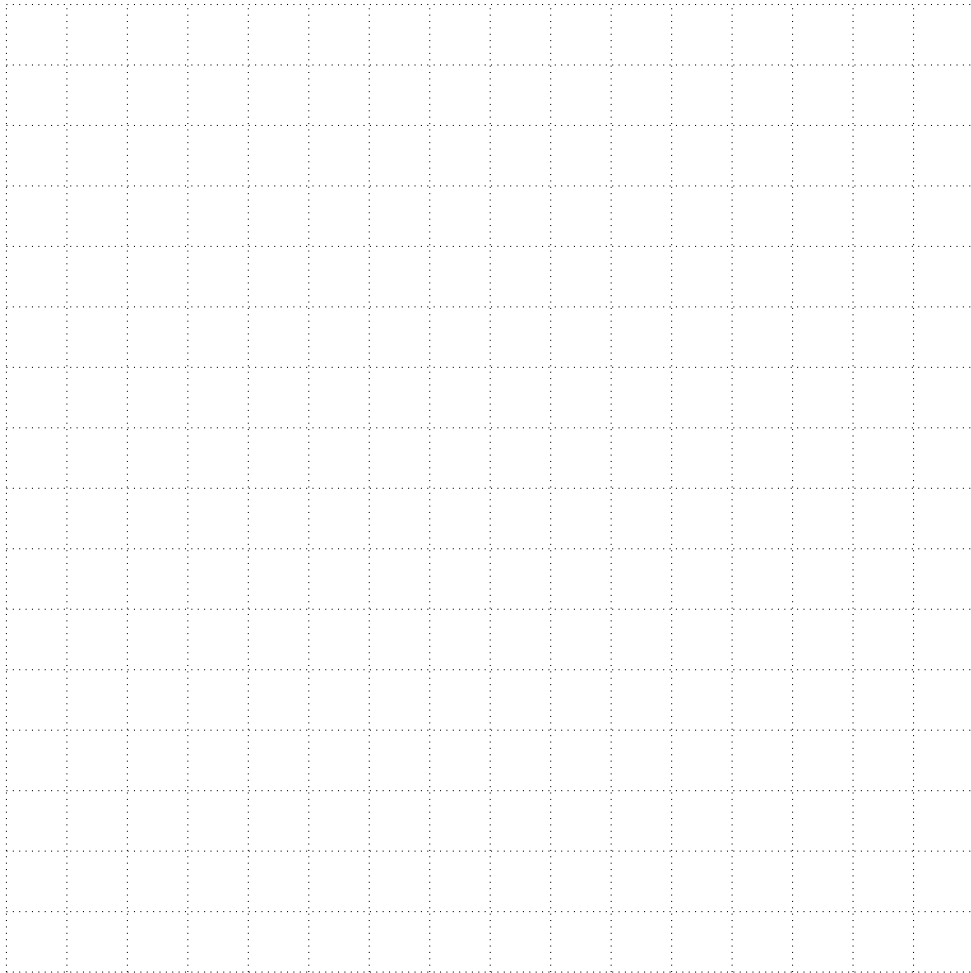
(2) (a) Verify, by differentiating and factoring, that  $f'(x) = 12x(x-1)^2$  and  $f''(x) = 12(x-1)(3x-1)$ .

(3) (b) Determine the intervals on which  $f(x)$  is increasing or decreasing.

(3) (c) Find all local maximum and minimum values of  $f$ , if any.

(2) (d) Determine the intervals on which  $f$  is concave up or concave down.

- (5) 12. Neatly sketch the graph of the function  $f$  defined in question (11) on the grid below. Use all your results from question (11).



- (8) 13. The space within a race track of length 2 km is to consist of a rectangle with a semi-circular area at each end. To what dimensions should the track be built to maximize the area of the rectangle? Justify why there is a maximum.