Math 1003 Final Examination, Fall 2010

Department of Mathematics and Statistics
University of New Brunswick

Please circle the name of your section/instructor.

1A–Dalkir	3A–Watmough	5A–Sankey	7A–Nanayakkara
2A–Dalkir	4A–Watmough	6A–Sankey	

Answer each question in the space provided. Use the reverse side for rough work. The marks for each question are shown in the left margin.
The exam is closed book, the use of calculators is NOT permitted.

Total Marks: 100 Time: 175 min

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1. Use the rules of differentiation to compute the derivatives of the following functions (do <u>not</u> simplify the answers):

(4) (a) $y = x^6 - \dfrac{3}{\sqrt{x}} + \dfrac{5}{x^2} - 2$

(4) (b) $y = \dfrac{e^{2x}}{1 - e^{x^2}}$

(4) (c) $g(t) = \ln(\cos t) - \sin(\ln t)$

(4) (d) $y = \arcsin\left(\dfrac{2}{x}\right)$ (i.e. $y = \sin^{-1}\left(\dfrac{2}{x}\right)$)

(4) (e) $y = (x+1)^{(1-x)}$

2. Use the rules of differentiation to compute the derivatives of the following functions. Please simplify your answers.

(3) (a) $f(x) = x\tan^{-1}\left(\frac{x}{2}\right) - \ln\left(x^2+4\right)$

(3) (b) $g(\theta) = \ln\sec^2\theta$

(3) (c) $p(x) = \sqrt{\frac{(x^2+x)}{x^2}}$

3. Compute these limits. Show your work. In particular, if a limit does not exist, briefly explain why.

(2) (a) $\lim\limits_{\theta\to\pi/2} \frac{1-\sin\theta}{1+\cos 2\theta}$

(2) (b) $\lim\limits_{x\to 0} \frac{3^x-1}{2^x-1}$

(2) (c) $\lim\limits_{t\to 0} \frac{t-3}{t^2-3}$

(8) 4. Find an equation for the line tangent to the curve defined implicitly by the relation

$$6x^2 + 3xy + 2y^2 + 17y = 6$$

at the point $(-1, 0)$.

(2) 5. Evaluate the expression $\tan\left(\cos^{-1} x\right)$.

6. Suppose f is continuous on $[1, 5]$ and differentiable on $(1, 5)$, with $f(1) = 8$, $f(5) = 4$ and $f'(3) = 1$. Answer each of the following, and justify your answers.

(3) (a) Does $f'(x) = -1$ for some x in $(1, 5)$?

(3) (b) Does $f'(x) = 0$ for some x in $(1, 5)$?

(3) (c) Could $f(x) > 8$ for some x in $[1, 5]$?

(3) (d) Does f have an absolute minimum on $[1, 5]$?

(8) 7. Sand falls from a conveyor belt at the rate of 10 $\mathrm{m}^3/\mathrm{min}$ onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast is the radius of the base of the pile increasing when the pile is 4 m high?. Note that the volume of a cone is $\frac{1}{3}\pi r^2 h$.

(2) 8. (a) Find and justify with limits all vertical or horizontal asymptotes (if any) for the graph of

$$y = \frac{x - 3x^2}{x^2 - 4}.$$

(3) (b) On the set of axes indicate the asymptotes and sketch a <u>possible</u> graph for the curve which indicates how the curve behaves near the asymptotes. Do <u>not</u> compute the derivatives of y.

(3) 9. Find a function f satisfying $f'(x) = \sin(\pi x/4) + \dfrac{1}{x}$ and $f(2) = 0$.

(4) 10. Find the area under the graph of $y = x^2 + 7$ and over the interval $1 \leq x \leq 2$.

11. Throughout this question, f is the function defined by $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$.

(2) (a) Verify, by differentiating and factoring, that $f'(x) = 12x(x-1)^2$ and $f''(x) = 12(x-1)(3x-1)$.

(3) (b) Determine the intervals on which $f(x)$ is increasing or decreasing.

(3) (c) Find all local maximum and minimum values of f, if any.

(2) (d) Determine the intervals on which f is concave up or concave down.

(5) 12. Neatly sketch the graph of the function f defined in question (11) on the grid below. Use all your results from question (11).

(8) 13. The space within a race track of length 2 km is to consist of a rectangle with a semi-circular area at each end. To what dimensions should the track be built to maximize the area of the rectangle? Justify why there is a maximum.