STUDENT'S NAME:____________________________ ID#:________________

PLEASE CIRCLE INSTRUCTOR AND SECTION:

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MATH 1003

Total Points = 100 Final Exam, December 13, 2011 TIME: 3 HOURS

Instructions: Calculators and other electronic devices are not permitted. Show your work, so that your answers are justified. If you need extra space, use the backs of the pages.

1. Find the derivative. Answers need not be simplified.

   (a) $x^2 \cos(\pi x)$ (4)

   (b) $\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt[4]{x^7}}$ (4)

   (c) 10^{t^2-t} (4)

   (d) $\dfrac{x \ln x}{x^3+1}$ (4)

   (e) $\left(\tan^{-1}\theta + 5\right)^4$ (4)

2. Answer (a) and (b) with regard to the curve described by the equation below. (8)

$$x^2 + xy - y^2 = 4$$

(a) Find the slope of the tangent line at the point $(2, 2)$.

(b) Find the equation of the tangent line at the point $(2, 2)$.

3. Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = (x-2)^{2x+5}$. Express your answer as a function of x. (6)

4. Find the indicated limits. If the limit is infinite, write "$+\infty$" or "$-\infty$". If the limit does not exist and is not infinite, write "DNE". You may use L'Hospital's Rule where appropriate. Justify your answers.

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}}$ (3)

(b) $\displaystyle\lim_{x\to 1} e^{x^3-x}$ (3)

(c) $\displaystyle\lim_{z\to 3} \frac{\sqrt{z+6}-z}{z^3-3z^2}$ (3)

(d) $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{x^2+1}}{x}$ (3)

5. Find all vertical and horizontal asymptotes for $y = \dfrac{5x-5}{x^4+x^2}$. Make sure to justify your answer with limits. (5)

6. A rock is thrown upward, on Mars, with initial velocity 19 m/s. Its height at time t is

$$h(t) = 19t - 1.9t^2.$$

(a) What is the acceleration of the rock at time t? (3)

(b) How fast is the rock going when it hits the ground? (5)

7. This question extends over two pages. Answer all parts of this question with regard to the graph of the function (15)

$$f(x) = \frac{x}{x^2 + 9}.$$

(a) State the domain of f.

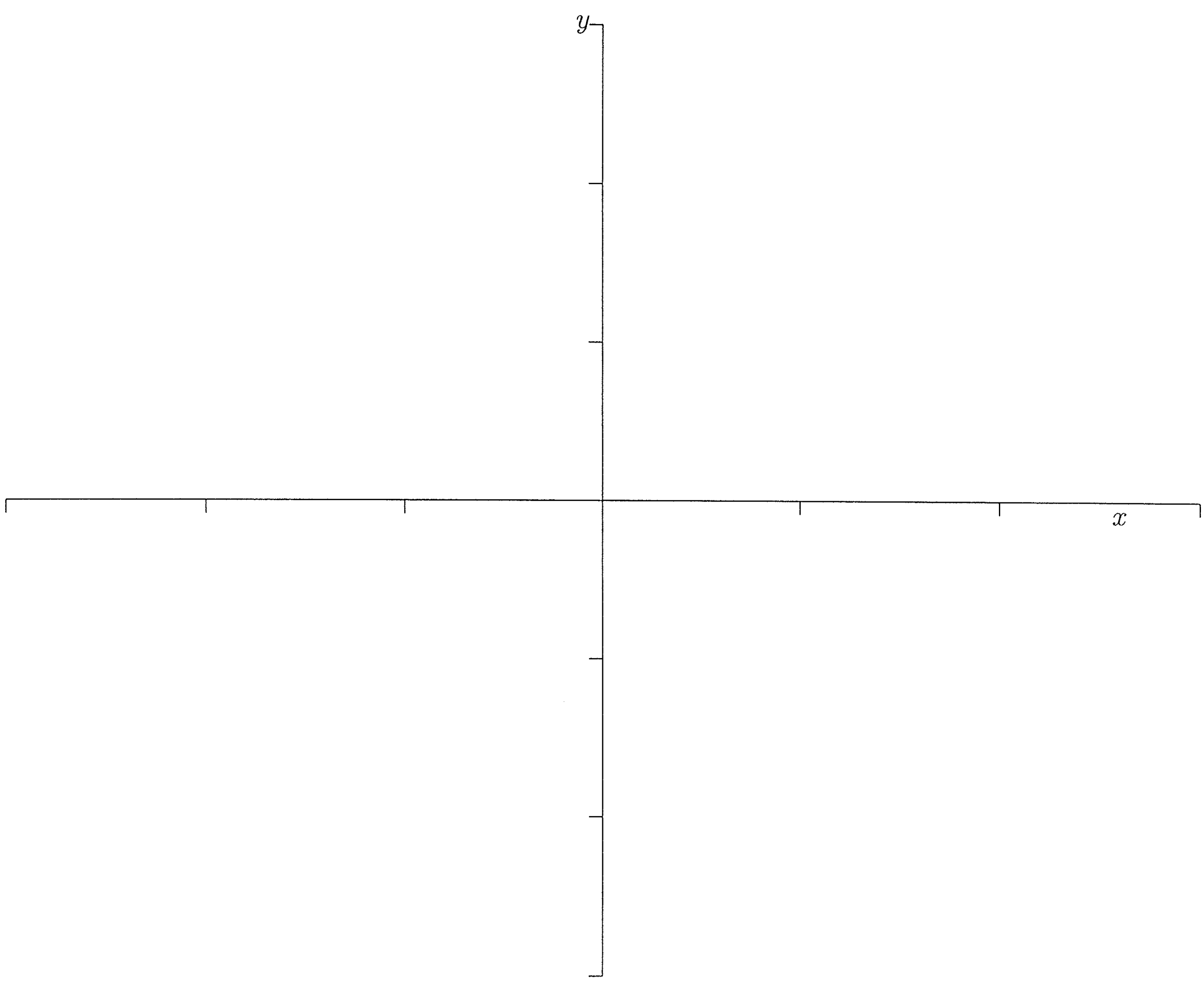
(b) Find the intervals on which f is increasing, and the intervals on which it is decreasing.

(c) Find the coordinates of any local extrema.

(d) Find the intervals on which the graph is concave up, and the intervals on which it is concave down.

(e) Find the coordinates of any points of inflection.

(f) Sketch the graph of $y = f(x)$, taking care to incorporate your answers to (a) through (e) above.

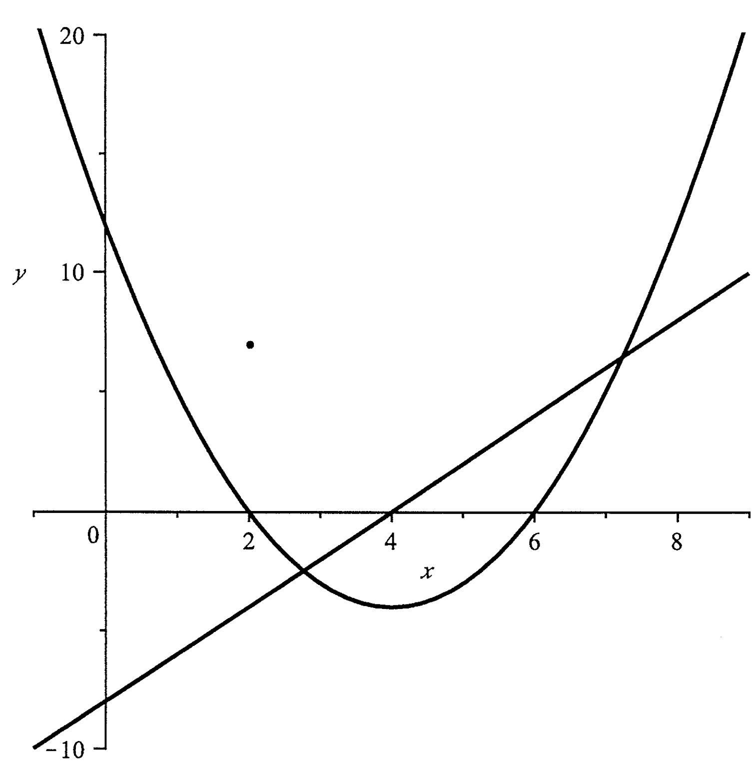


8. A particle is moving along the circle $x^2 + y^2 = 25$. If $\dfrac{dx}{dt} = 2$ cm/s when the particle reaches the point $(3, -4)$,

(a) Find $\dfrac{dy}{dt}$ at that point. (5)

(b) Sketch the circle and indicate in which direction the particle is travelling. (1)

9. Below are the graphs of $y = f'(x)$ and $y = f''(x)$ for a function f. (You should be able to deduce which is which.) Sketch the graph of $y = f(x)$, given that it passes through the point $(2, 7)$. (3)



10. A cylindrical can of largest possible volume is to be constructed, from 600 cm^2 of material. (Some of the material will be used for the top and the bottom of the can.) Find the dimensions of the can. (7)

11. Consider this statement: (5)

If $f(1) > 0$ and $f(3) < 0$, then $f(c) = 0$ for some number c.

Is this statement: (a) Always true? (b) True under certain conditions? (c) Never true? Justify your anwer.

12. Sketch a function with the indicated properties, or explain why it is not possible.

$f'(x) > 0$ on $(-\infty, \infty)$ and $f''(x) > 0$ on $(-\infty, 1) \cup (3, \infty)$ and $f''(x) < 0$ on $(1, 3)$. (5)