DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION
APRIL 2000

TIME: 3 HOURS
TOTAL POINTS = 100

- NO CALCULATORS, NOTES OR BOOKS.

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PART I: Perform each integration. Each is worth 2 marks. You must obtain 15 out of 22 marks in PART I to pass Math 1013.

1. $\int (e^x + 4x)dx$
2. $\int \left(x + \frac{1}{x}\right)^2 dx$
3. $\int \sin(3x)\, dx$
4. $\int \frac{1}{x^2+1}\, dx$
5. $\int \frac{3x^2}{x^3-1}\, dx$
6. $\int \sin^2 x \cos x\, dx$
7. $\int_1^2 \frac{1}{x+1}\, dx$
8. $\int x \cos x\, dx$
9. $\int \cot x\, dx$
10. $\int \sec 2x\, dx$
11. $\int 3^x\, dx$

VALUES

PART II: Do each question. Show your work. Circle or box your final answer.

1. Do each integration:

(5) (a) $\int \frac{x-1}{x(x^2+2x+1)}\, dx$

(5) (b) $\int_0^{\pi/2} \cos 2x \cos 3x\, dx$

(5) (c) $\int \sec^3 x \tan^3 x \, dx$

2. Let R be the region bounded by $y = x + 1$ and $y = (x-1)^2$.

(4) (a) Sketch the region R.

(5) (b) Find the area of R.

(5) 3. Let R be the region bounded by $y^2 = x$ and $x = 2y$. Find the volume of the solid obtained by rotating R about the y-axis.

(4) 4. (a) Compute the value of the definite integral $\int_{-1}^{1} (x+2)dx$ from the definition; i.e., as a limit of a Riemann sum.

(4) (b) Find the Taylor polynomial of degree 3 for $\cos x$ expanded about $a = 0$.

(4) (c) Use the result of part (b) to approximate $\cos(0.1)$.

5. For each improper integral, compute its value if it converges.

(4) (a) $\displaystyle\int_1^2 \frac{dx}{x^2-1}$

(4) (b) $\displaystyle\int_e^\infty \frac{dx}{x(\ln x)^2}$

(5) 6. Consider the integral $I = \displaystyle\int_0^{2\pi} x \sin x \, dx$.

Do either part (a) or part (b) below, but not both. State which part you are doing.

(a) Approximate I by the left-hand rule with 4 subintervals of equal length; **OR**

(b) Approximate I by the trapezoid rule with 4 subintervals of equal length.

(4) 7. (a) Solve the initial value problem

$$xy' = \frac{(1+y^2)}{x} \; ; \quad y\left(\frac{4}{\pi}\right) = -1.$$

(4) (b) Find the general solution of $x^2y' + 2xy = e^{3x}$.

8. Each question below concerns complex numbers.

(a) Write in standard form:

(2) (i) $\displaystyle\frac{x - iy}{x + iy}$

(2) (ii) $7e^{\frac{7\pi}{6}i}$ Note: $e^{i\theta} = \cos\theta + i\sin\theta$

(b) Write in polar form:

(2) (i) $\displaystyle -\frac{7}{2} + \frac{7}{2}\, i$

(2) (ii) $-4 - 3i$

(4) (iii) Use DeMoivre's formula to prove that

$$(\overline{z})^n = \overline{(z^n)}.$$

(4) (iv) Find the cube roots of $27i$.