NAME:________________________________ STUDENT ID #:______________

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION
APRIL 2001

TIME: $2\frac{1}{2}$ HOURS
TOTAL POINTS = 100

INSTRUCTIONS:

1. Note that you have choices in questions 2 and 8.
2. Calculators are not allowed and so are not needed. Exact values, like $\ln 2, \pi, 2^{3.6}$ are perfectly good.
3. A short table of integrals has been supplied, though you may find it just as fast to do problems from scratch. Please indicate which items from the table you do use.

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(24) 1. Evaluate each integral (3 points each).

(a) $\displaystyle\int \frac{dx}{e^x}$

(b) $\displaystyle\int_1^4 \frac{dx}{2x-1}$

(c) $\displaystyle\int \frac{x^3 - 2\sqrt{x}}{x}\, dx$

(d) $\displaystyle\int \frac{\cos x}{(1+\sin x)^2}\, dx$

(e) $\displaystyle\int \sec^2 3x\, dx$

(f) $\displaystyle\int \frac{x+4}{x^2+4}\, dx$

(g) $\displaystyle\int \tan\left(\frac{x}{2}\right)\, dx$

(h) $\displaystyle\int_1^e \ln x\, dx$

(20) 2. Evaluate just **four** (4) of the following integrals (5 points each).

(a) $\displaystyle\int \frac{dx}{(x^2+9)^{3/2}}$

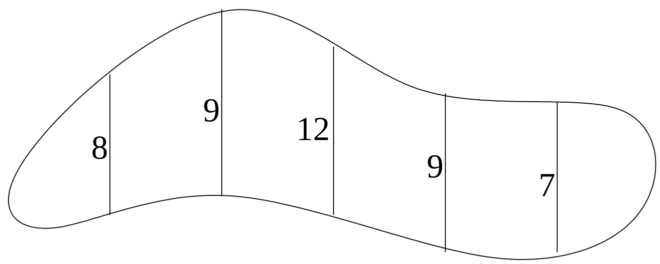
(b) $\displaystyle\int \frac{2x^2+x+16}{(x-2)(x^2+9)}\, dx$

(c) $\displaystyle\int \tan^3 x \sec^4 x \, dx$

(d) $\displaystyle\int x^2 \cos(5x) \, dx$

(e) $\displaystyle\int \arctan x \, dx$ (i.e., $\displaystyle\int \tan^{-1} x \, dx$)

(6) 3. The width (in km) of an oil spill in the Bay of Fundy was measured at 1/2 km. intervals and found to be as shown here:



Find the approximate area of the spill in two ways:

(a) using a Riemann sum;

(b) using Simpson's rule.

(10) 4. (a) Find the general solution to $x\dfrac{dy}{dx} - 2y = x^3$.

(b) Solve the initial value problem

$$\begin{aligned} xy\frac{dy}{dx} &= \ln x \\ y(1) &= 2. \end{aligned}$$

(7) 5. (a) Approximate $f(x) = \cos(x)$ by a Taylor polynomial of degree 2, centred at $a = 0$.

(b) Use the polynomial in part (a) to estimate $\cos(2^\circ)$. (Do not attempt to give your answer in decimal form.)

(7) 6. Determine whether the following improper integrals converge or diverge. If convergent, give the actual value of the integral.

(a) $\displaystyle\int_1^5 \frac{dx}{\sqrt{x-1}}$

(b) $\displaystyle\int_2^\infty \frac{dx}{x^3}$

(8) 7. This question concerns complex numbers.

(a) If $z = x + iy$, give the real and imaginary parts of $\dfrac{(\overline{z})}{i}$.

(b) Find all complex solutions to the equation

$$z^3 + 64 = 0.$$

(18) 8. Do just **three** (3) of these problems (worth 6 points each). Not all sections did the same applications, so choose sensibly.

(a) Sketch the graphs of $y^2 = x$ and $y = x - 2$ and find the area enclosed by them.

(b) The region bounded by $y = \dfrac{1}{\sqrt{1-x}}$ and $y = 1$, for $0 \le x \le \dfrac{1}{2}$, is revolved about the x-axis. Calculate the resulting volume.

(c) A bacterial culture starts with 1000 bacteria and grows at a rate proportional to the current number of bacteria. After 2 hours, the population is 9000. What is the population after t hours?

(d) Find the length of the graph of $y = \cosh x$, for $0 \le x \le \ln 2$.

(e) Starting from the origin, an object moves along the x-axis so that at time t its velocity is

$$v(t) = te^{-t} \qquad (t \ge 0).$$

Where is the object located when $t = 2$?

(f) A force of $40N$ is required to hold a spring that has been stretched from its natural length of $0.10m$ to $0.15m$. How much work is done in stretching the spring from $0.15m$ to $0.18m$? (Recall: according to Hooke's Law, the force due to a spring is proportional to its displacement from its natural length.)