

NAME: \_\_\_\_\_

STUDENT ID #: \_\_\_\_\_

**DEPARTMENT OF MATHEMATICS & STATISTICS**

**MATH 1013**

FINAL EXAMINATION  
APRIL 2001

TIME:  $2\frac{1}{2}$  HOURS  
TOTAL POINTS = 100

**INSTRUCTIONS:**

1. Note that you have choices in questions 2 and 8.
2. Calculators are not allowed and so are not needed. Exact values, like  $\ln 2$ ,  $\pi$ ,  $2^{3.6}$  are perfectly good.
3. A short table of integrals has been supplied, though you may find it just as fast to do problems from scratch. Please indicate which items from the table you do use.

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- (24) 1. Evaluate each integral (3 points each).

(a)  $\int \frac{dx}{e^x}$

(b)  $\int_1^4 \frac{dx}{2x-1}$

(c)  $\int \frac{x^3 - 2\sqrt{x}}{x} dx$

(d)  $\int \frac{\cos x}{(1 + \sin x)^2} dx$

(e)  $\int \sec^2 3x dx$

(f)  $\int \frac{x+4}{x^2+4} dx$

(g)  $\int \tan\left(\frac{x}{2}\right) dx$

(h)  $\int_1^e \ln x dx$

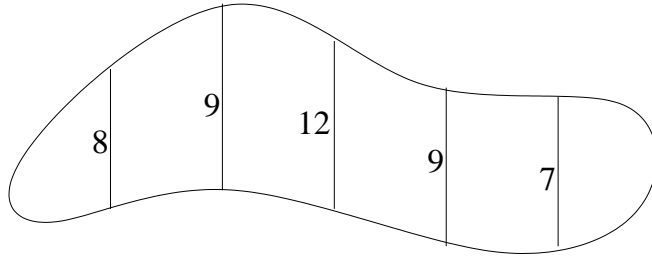
- (20) 2. Evaluate just **four** (4) of the following integrals (5 points each).

(a)  $\int \frac{dx}{(x^2+9)^{3/2}}$

(b)  $\int \frac{2x^2+x+16}{(x-2)(x^2+9)} dx$

- (c)  $\int \tan^3 x \sec^4 x \, dx$   
 (d)  $\int x^2 \cos(5x) \, dx$   
 (e)  $\int \arctan x \, dx$  (i.e.,  $\int \tan^{-1} x \, dx$ )

- (6) 3. The width (in km) of an oil spill in the Bay of Fundy was measured at 1/2 km. intervals and found to be as shown here:



Find the approximate area of the spill in two ways:

- (a) using a Riemann sum;  
 (b) using Simpson's rule.
- (10) 4. (a) Find the general solution to  $x \frac{dy}{dx} - 2y = x^3$ .  
 (b) Solve the initial value problem

$$\begin{aligned} xy \frac{dy}{dx} &= \ln x \\ y(1) &= 2. \end{aligned}$$

- (7) 5. (a) Approximate  $f(x) = \cos(x)$  by a Taylor polynomial of degree 2, centred at  $a = 0$ .  
 (b) Use the polynomial in part (a) to estimate  $\cos(2^\circ)$ . (Do not attempt to give your answer in decimal form.)
- (7) 6. Determine whether the following improper integrals converge or diverge. If convergent, give the actual value of the integral.

(a)  $\int_1^5 \frac{dx}{\sqrt{x-1}}$   
 (b)  $\int_2^\infty \frac{dx}{x^3}$

- (8) 7. This question concerns complex numbers.
- (a) If  $z = x + iy$ , give the real and imaginary parts of  $\frac{(\bar{z})}{i}$ .  
 (b) Find all complex solutions to the equation

$$z^3 + 64 = 0.$$

- (18) 8. Do just **three** (3) of these problems (worth 6 points each). Not all sections did the same applications, so choose sensibly.

- (a) Sketch the graphs of  $y^2 = x$  and  $y = x - 2$  and find the area enclosed by them.
- (b) The region bounded by  $y = \frac{1}{\sqrt{1-x}}$  and  $y = 1$ , for  $0 \leq x \leq \frac{1}{2}$ , is revolved about the  $x$ -axis. Calculate the resulting volume.
- (c) A bacterial culture starts with 1000 bacteria and grows at a rate proportional to the current number of bacteria. After 2 hours, the population is 9000. What is the population after  $t$  hours?
- (d) Find the length of the graph of  $y = \cosh x$ , for  $0 \leq x \leq \ln 2$ .
- (e) Starting from the origin, an object moves along the  $x$ -axis so that at time  $t$  its velocity is

$$v(t) = te^{-t} \quad (t \geq 0).$$

Where is the object located when  $t = 2$ ?

- (f) A force of  $40N$  is required to hold a spring that has been stretched from its natural length of  $0.10m$  to  $0.15m$ . How much work is done in stretching the spring from  $0.15m$  to  $0.18m$ ? (Recall: according to Hooke's Law, the force due to a spring is proportional to its displacement from its natural length.)