DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION
APRIL 2002

TIME: 3 HOURS
TOTAL POINTS = 100

INSTRUCTIONS:

(a) You must show all calculations for full marks.

(b) Calculators **are not** permitted. (Huge calculations often mean you are on the wrong track!)

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MARKS

1. Evaluate the following integrals:

(3) (a) $\displaystyle\int \frac{1}{e^x}\,dx$

(3) (b) $\displaystyle\int \frac{3}{2+5x}\,dx$

(3) (c) $\displaystyle\int \frac{8}{\cos x}\,dx$

(3) (d) $\displaystyle\int \frac{\cos x}{\sin^2 x}\,dx$

(3) (e) $\displaystyle\int \tan 2x\,dx$

(4) (f) $\displaystyle\int \frac{1}{4x^2+1}\,dx$

(4) (g) $\displaystyle\int \frac{x^2+1}{x+1}\,dx$

(4) (h) $\displaystyle\int x\ \ln 3x\,dx$

(4) (i) $\displaystyle\int \frac{5-x}{x^2-x-2}\,dx$

(4) (j) $\displaystyle\int \sin^2 x\ \cos^2 x\,dx$

2. Decide whether the following improper integrals converge or diverge. If the integral converges, find its value.

(4) (a) $\displaystyle\int_0^\infty \frac{x}{1+x^2}\,dx$

(4) (b) $\displaystyle\int_2^6 \frac{1}{\sqrt{x-2}}\,dx$

(3) 3. Let $f(x) = \begin{cases} x & \text{if } \ 0 \le x \le 1 \\ 1 & \text{if } \ 1 \le x \le 2 \end{cases}$. Compute $\displaystyle\int_0^2 f(x)dx$.

(4) 4. At $t = 0$, a car is at rest. The velocity in m/s at one second intervals is given by

Time (seconds)	0	1	2	3	4	5	6
Velocity (m/s)	0	1	4	10	12	8	0

Use either the Trapezoidal rule or Simpson's rule to approximate the distance the car travels between $t = 0$ seconds and $t = 6$ seconds. Clearly state which method you are using.

(2) 5. (a) Sketch the region R bounded by $y = x + 2$ and $y = x^2$.

(3) (b) Find the area of the region R in part (a).

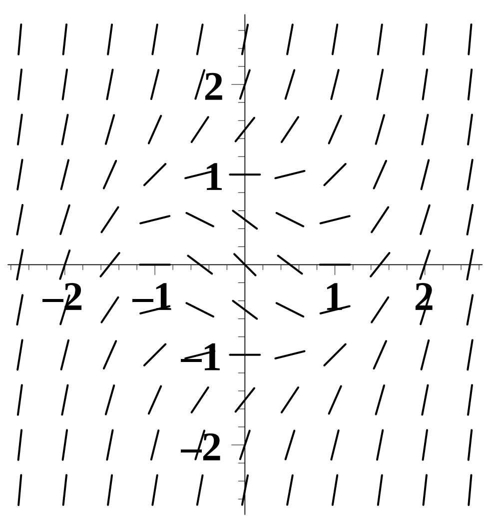
(c) Set up, but ***do not evaluate***, integrals for the volume obtained when the region R is rotated about

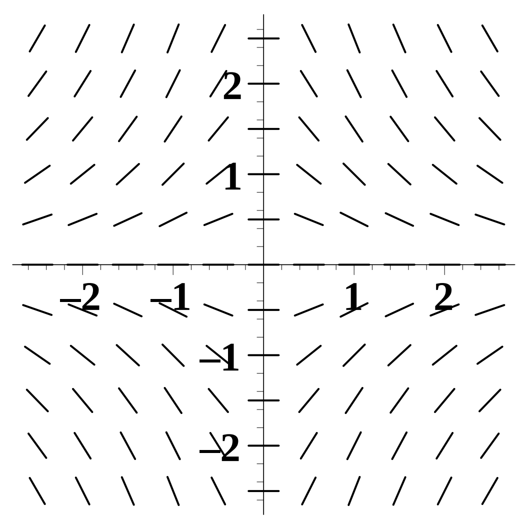
(2) (i) the x-axis;

(2) (ii) the line $y = 5$;

(2) (iii) the line $x = -1$.

(3) 6. (a) Which of the following two graphs represents the direction field (also known as the slope field) of the differential equation $y' = x^2 + y^2 - 1$? Briefly justify your answer.





(2) (b) Suppose the differential equation from (a) (i.e., $y' = x^2 + y^2 - 1$) has initial value $y(0) = 1$. Without solving the differential equation, sketch the solution curve directly on the appropriate graph.

7. Solve the following initial value problems:

(4) (a) $y' + 3y = e^{-3x}$, $y(0) = 4$;

(4) (b) $y' = (y^2 + 1)x$, $y(-2) = 0$.

(4) 8. (a) Find the Taylor polynomial of degree 2 for $y = \sin x + \cos x$ expanded about $x = 0$.

(b) Approximate $\sin(0.1)1 + \cos(0.1)$ using the result of (a).

9. **Complex Numbers.**

(3) (a) Simplify and express in the form $a + bi$, where a and b are real numbers.

(i) $\overline{1 - 2i} + |1 - 2i|$

(ii) $\dfrac{10 - 5i}{2 + i} + i$

(iii) $2\left(e^{\frac{\pi}{8}i}\right)^2$ (Note: $e^{\theta i} = \cos\theta + i\sin\theta$)

(2) (b) Write in polar form: $(-1 + i)^4$.

(5) (c) Find all complex solutions of $z^4 = -1$.

(12) 10. **DO TWO (2) OF THE FOLLOWING FOUR QUESTIONS:**

(a) A piece of rope is to be used to lift a bucket from the ground to the top of a 10 meter building. The bucket and its content weighs 5 kilograms. The rope weighs $1/2$ kg per meter. How much work is done lifting the bucket from the ground to the top of the building?

(b) An end of a spring moves so that at time t the acceleration of a mass attached to it is given by $a(t) = 18\sin 3t\ m/s^2$. At $t = 0$ the velocity is $-6\ m/sec$ and the position is $3\ m$. What is the position of the object at time $t = \pi/6\ sec$?

(c) 100 rabbits are introduced to an island, on which there were previously no rabbits. Three years later there were 500 rabbits. Assuming the rate of growth is proportional to the number of rabbits, how many rabbits are there after t years?

(d) The Easter Bunny has 200 litres of sugar syrup left over from her easter candy making. The sugar syrup has .2 kg/litre of sugar. Pure water is added to the tank at a rate of 10 litres/minute, and the well-mixed solution is drained at the same rate. How much sugar is in the tank 20 minutes later?