

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION
APRIL 2003

TIME: 3 HOURS
TOTAL POINTS = 90

INSTRUCTIONS:

- (a) You must show all calculations for full marks.
- (b) Calculators **are not** permitted. (Huge calculations often mean you are on the wrong track!)

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MARKS

1. Evaluate the following integrals:

(3) (a) $\int \frac{e^x}{\sqrt{1+e^x}} dx$

(3) (b) $\int_0^2 \frac{1}{4+x^2} dx$

(3) (c) $\int \sec 2x dx$

(3) (d) $\int \frac{x+3}{x-3} dx$

(4) (e) $\int \sin^3 x \cos^2 x dx$

(4) (f) $\int x \sin 2x dx$

(4) (g) $\int \frac{3x-10}{x^2-7x+12} dx$

(4) (h) $\int_0^2 \sqrt{4-x^2} dx$

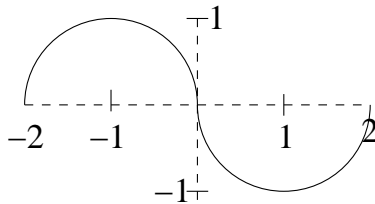
2. Decide whether the following improper integrals converge or diverge. If the integral converges, find its value.

(3) (a) $\int_0^\infty e^{-x} dx$

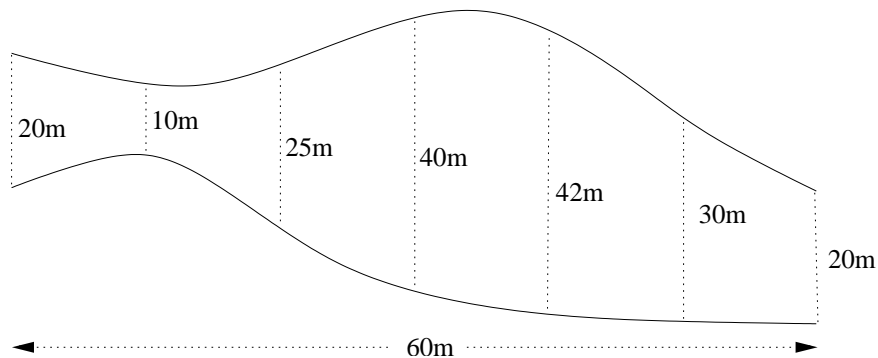
(3) (b) $\int_0^5 \frac{1}{x^2} dx$

(4) 3. Find b such that the average value of $y = x^2$ on $[0, b]$ ($b > 0$) is 3.

- (2) 4. Let f be the function whose graph consists of two semi-circles, as below:



- (a) What is $\int_0^2 f(x) dx$?
- (b) What is $\int_{-2}^2 f(x) dx$?
- (4) 5. A surveyor wishes to approximate the area of a section of river. He measures its width every 10 meters, as indicated on the diagram. Use either trapezoidal rule or Simpson's rule to find an approximation for the area.



6. Solve the following problems:
- (4) (a) $\frac{dy}{dx} - 2y = 3e^{2x}$;
- (4) (b) $\frac{dy}{dx} = 3x^2y$, $y(0) = 2$.
- (4) 7. (a) Find the Taylor polynomial of degree 2 for $f(x) = \sqrt{4+x}$ expanded about 0.
 (b) Approximate $\sqrt{3.9}$ using the result of (a).
8. Consider the region R bounded by $y = \frac{x^3}{2}$, $y = 0$ and $x = 2$.
- (1) (a) Sketch the region.
- (2) (b) Find the area of R .
- (c) Set up, but **do not evaluate**, integrals for the volume obtained when the region is rotated about
- (2) (i) the x -axis;
- (2) (ii) the y -axis;
- (2) (iii) the line $y = -1$.

9. Complex Numbers.

- (a) Simplify and express in the form $a + bi$, where a and b are real numbers.
- (1) (i) $\sqrt{-3} \cdot \sqrt{-12}$
- (1) (ii) $|4 - 3i| + \overline{4 - 3i}$
- (1) (iii) $\frac{3 - 4i}{3 + 4i}$
- (2) (iv) $(e^{\pi i})^{1/2}$
- (2) (v) $(1 + i)^{10}$
- (3) (b) Find all complex solutions of $z^3 = -8$.

(15) 10. **DO 3 OF THE FOLLOWING 5 PROBLEMS:**

- (a) A bacteria culture starts with 600 bacteria and grows at a rate proportional to its size. After 3 hours, there are 4800 bacteria.
- (i) Find an expression for the number of bacteria after t hours.
- (ii) Find the number of bacteria after 4 hours.
- (b) Evaluate $\int e^{2x} \sin x \, dx$
- (c) There are many regions enclosed between $y = \sin x$ and $y = \frac{1}{2}$. Find the area of one of them.
- (d) When a certain spring is stretched, it pulls back with a force proportional to the distance through which it is stretched. The work required to stretch the spring from its unstretched length to $0.03 \, m$ beyond its unstretched length is 9 joules. How much work is needed to stretch the spring $0.04 \, m$ beyond its unstretched length?
- (e) A tank contains 1000 litres of a solution consisting of $100 \, kg$ of salt dissolved in water. Pure water is pumped into the tank at $5 \, l/s$ and the well stirred mixture is pumped out at the same rate. How long will it be until only $10 \, kg$ of salt remains in the tank.