

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION
APRIL 2004

TIME: 3 HOURS
TOTAL POINTS = 100

INSTRUCTIONS:

- (a) You must show all your work for full marks.
- (b) Calculators are not needed and are not permitted.

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MARKS

1. Evaluate the following integrals:

- (2) (a) $\int \frac{dx}{e^x}$
- (3) (b) $\int_5^6 \frac{2}{4-x} dx$
- (3) (c) $\int \frac{x^3 - 5\sqrt{x} + 3}{x} dx$
- (4) (d) $\int \frac{\arcsin(2x)}{\sqrt{1-4x^2}} dx$
- (6) (e) $\int_0^\pi (x+1) \cos x dx$
- (6) (f) $\int \frac{1-3x^2}{x^3+2x^2+x} dx$
- (6) (g) $\int_0^{\pi/4} \tan^3 x \sec^3 x dx$

- (4) 2. (a) Let $f(x) = \begin{cases} \sqrt{1-x^2} & 0 \leq x \leq 1 \\ 2x-2 & 1 < x \leq 3 \end{cases}$.

Sketch the graph of $f(x)$ and find the value of $\int_0^3 f(x) dx$ without finding any antiderivatives.

- (4) (b) The function $C(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt$ is used in the theory of diffraction of light waves. Find one solution to $\frac{dC}{dx} = 0$.

3. Determine if these improper integrals are convergent or divergent. If convergent, find the value.

- (4) (a) $\int_0^\infty x e^{-x^2} dx$

(4) (b) $\int_{-2}^{-1} \frac{1}{(1+x)^{1/3}} dx$

(4) 4. (a) Use Simpson's rule with $n = 4$ to find an approximate value for $\int_0^2 4^{-x} dx$.

(2) (b) Find the exact value of the integral in part (a).

5. Solve the following differential equations:

(4) (a) $\frac{dy}{dx} = \frac{y}{x^3}$, $y(1) = 1$

(4) (b) $y' - 3x^2y = 2xe^{x^3}$

(5) (c) $4y'' + 4y' + y = 0$, $y(0) = -1$, $y'(0) = 1$

(4) (d) $y'' - y' + y = 0$

(5) 6. (a) Sketch the region R bounded by the curves $y = 4x + 4$ and $y = x^2 - 1$ and find the area of R .

(3) (b) Set up, but do not evaluate, an integral representing the volume when the region bounded by $y = \ln x$, $x = 0$, $y = 0$ and $y = 2$ is revolved about the y -axis.

(3) 7. (a) Find the Taylor polynomial of degree 2 for $f(x) = \sec x$ expanded about $x = 0$.

(2) (b) Use part (a) to give an approximate value for $\sec(0.1)$.

(3) (c) Find the Taylor series for $y = e^{x/2}$ about $x = 1$.

(15) 8. **DO ANY THREE OF THE FOLLOWING:**

(a) Find the arc length of $y = \frac{x^4}{16} + \frac{1}{2x^2}$ for $1 \leq x \leq 2$.

(b) The half-life of strontium 90 is 25 years.

(i) Starting with 18 mg., find the formula which determines the amount left after t years.

(ii) How long will it take until only 2 mg. remain?

(c) Evaluate: $\int \frac{x^2}{\sqrt{4-x^2}} dx$.

(d) A solid is built on the circle $x^2 + y^2 = 4$ as base. Each cross-section perpendicular to the x -axis is a square. Find its volume.

(e) Solve: $y' = y(1 - y)$ if $y(0) = 2$.