

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION
DECEMBER 2004

TIME: 3 HOURS
TOTAL POINTS = 100

- NO CALCULATORS, NOTES OR BOOKS.

1. Perform each integration. Each is worth 3 marks. **Do not simplify your answer.**

(a) $\int_0^1 \left(x + \frac{1}{2}\right) dx$

(b) $\int \left(\frac{x+1}{x^2}\right)^2 dx$

(c) $\int e^{3x} dx$

(d) $\int_0^{\pi/4} \tan x dx$

(e) $\int 3x\sqrt{1+x^2} dx$

(f) $\int \sin x \cos^2 x dx$

(g) $\int \sec x dx$

(h) $\int x \cos x dx$

VALUES

2. Do each integration:

(4) (a) $\int \frac{\sqrt{2-x^2}}{x} dx$

(4) (b) $\int (\csc^2 t \cot^2 t) dt$

(4) (c) $\int \frac{x+1}{x^3-4x} dx$

3. Let R be the region bounded by $y = e^x$ and $y = e^{-x}$ from $x = -1$ to $x = 0$.

(4) (a) Sketch the region R .

(5) (b) Find the area of R .

(5) 4. Find the volume of the solid obtained by rotating the region between $y = x^2$ and $y = 4x$ about the y -axis.

5. Compute the value of each improper integral, if it converges.

(4) (a) $\int_{-1}^1 \frac{1}{1-x} dx$

- (4) (b) $\int_0^{\infty} \frac{x}{(x^2 + 1)^{3/2}} dx$
- (4) 6. Compute the value of $\int_0^4 (x + 1) dx$ directly from its definition as a limit of Riemann sum. You may not use the Fundamental Theorem of Calculus.
- (4) 7. (a) Find the Taylor polynomial of degree 2 for $f(x) = xe^x$ expanded about $a = 0$.
- (4) (b) Use the result of part (a) to estimate the value of $f(0.1)$.
- (4) 8. Approximate the value of $I = \int_0^4 xe^x dx$ using the trapezoid rule with 4 equal subintervals. Do not attempt to express your answer as a decimal.
- (4) 9. (a) Find the general solution of $x \frac{dy}{dx} = \tan y$.
- (5) (b) Solve the initial value problem $y' + y = e^{-2x}$, $y(0) = 1$.
- (4) (c) A radioactive material has half-life 4000 years. Find how much of a 2 gm. sample remains after 100 years. Do not attempt to express your answer as a decimal.
- (d) Find the general solutions of the differential equations:
- (4) (i) $2y'' + 4y' + 2y = 0$;
- (4) (ii) $y'' + y' + y = 0$.
- (5) 10. Find the arc-length of the curve $y = \ln x$ from $x = 1$ to $x = 4$.