DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1013

FINAL EXAMINATION — TIME: 3 HOURS
DECEMBER 2004 — TOTAL POINTS = 100

- NO CALCULATORS, NOTES OR BOOKS.

1. Perform each integration. Each is worth 3 marks. **Do not simplify your answer.**

   (a) $\displaystyle\int_0^1 \left(x + \frac{1}{2}\right)\, dx$

   (b) $\displaystyle\int \left(\frac{x+1}{x^2}\right)^2\, dx$

   (c) $\displaystyle\int e^{3x}\, dx$

   (d) $\displaystyle\int_0^{\pi/4} \tan x\, dx$

   (e) $\displaystyle\int 3x\sqrt{1+x^2}\, dx$

   (f) $\displaystyle\int \sin x \cos^2 x\, dx$

   (g) $\displaystyle\int \sec x\, dx$

   (h) $\displaystyle\int x \cos x\, dx$

VALUES

2. Do each integration:

(4) (a) $\displaystyle\int \frac{\sqrt{2-x^2}}{x}\, dx$

(4) (b) $\displaystyle\int (\csc^2 t \cot^2 t)\, dt$

(4) (c) $\displaystyle\int \frac{x+1}{x^3 - 4x}\, dx$

3. Let R be the region bounded by $y = e^x$ and $y = e^{-x}$ from $x = -1$ to $x = 0$.

(4) (a) Sketch the region R.

(5) (b) Find the area of R.

(5) 4. Find the volume of the solid obtained by rotating the region between $y = x^2$ and $y = 4x$ about the y-axis.

5. Compute the value of each improper integral, if it converges.

(4) (a) $\displaystyle\int_{-1}^1 \frac{1}{1-x}\, dx$

(4) (b) $\displaystyle\int_0^\infty \frac{x}{(x^2+1)^{3/2}}\,dx$

(4) 6. Compute the value of $\displaystyle\int_0^4 (x+1)\,dx$ directly from its definition as a limit of Riemann sum. You may <u>not</u> use the Fundamental Theorem of Calculus.

(4) 7. (a) Find the Taylor polynomial of degree 2 for $f(x) = xe^x$ expanded about $a = 0$.

(4) (b) Use the result of part (a) to estimate the value of $f(0.1)$.

(4) 8. Approximate the value of $I = \displaystyle\int_0^4 xe^x\,dx$ using the trapezoid rule with 4 equal subintervals. Do <u>not</u> attempt to express your answer as a decimal.

(4) 9. (a) Find the general solution of $x\,\dfrac{dy}{dx} = \tan y$.

(5) (b) Solve the initial value problem $y' + y = e^{-2x},\ y(0) = 1$.

(4) (c) A radioactive material has half-life 4000 years. Find how much of a 2 gm. sample remains after 100 years. Do <u>not</u> attempt to express your answer as a decimal.

(d) Find the general solutions of the differential equations:

(4) (i) $2y'' + 4y' + 2y = 0$;

(4) (ii) $y'' + y' + y = 0$.

(5) 10. Find the arc-length of the curve $y = \ln x$ from $x = 1$ to $x = 4$.