

**DEPARTMENT OF MATHEMATICS & STATISTICS**  
**MATH 1013**

FINAL EXAM

NAME.....

APRIL 20, 2005

STUDENT I.D......

**INSTRUCTOR (Please Circle):**

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**SHOW YOUR WORK. USE THE BACKS OF THE PAGES IF NECESSARY.**  
**NO CALCULATORS, NOTES, TEXTS ALLOWED.**

MARKS

- (4) 1. Use the definition of the definite integral as the limit of a Riemann sum to compute  $\int_{-1}^3 4x dx$ . **Do not** use the Fundamental Theorem of calculus.

2. Evaluate each integral:

(3) (a)  $\int \tan x dx$

(3) (b)  $\int_0^{\frac{\pi}{3}} \frac{\sin x}{1 + \cos x} dx$

(3) (c)  $\int \frac{x^2}{x+3} dx$

(4) (d)  $\int x e^{-x} dx$

(4) (e)  $\int \sec^4 t dt$

(5) (f)  $\int \frac{3x^2 + 5x - 4}{(x-1)(x+1)^2} dx$

(4) (g)  $\int \frac{1}{(16-x^2)^{\frac{3}{2}}} dx$

3. Determine if the improper integrals converge or diverge. If it converges, determine its value.

(4) (a)  $\int_0^2 \frac{1}{(x-1)^2} dx$

(4) (b)  $\int_0^{\infty} x e^{-x^2} dx$

(4) 4. Find the total area between  $y = \sin x$  and the  $x$ -axis from  $x = 0$  to  $x = \frac{3\pi}{2}$

(4) 5. Set up, **but do not evaluate**, an integral for the arc-length of the curve  $y = \ln x$  from  $x = 0$  to  $x = 1$ .

6. Consider the region bounded by the parabola  $y = 4x - x^2$  and the line  $y = x$ . Set up, but do not evaluate, an integral for
- (4) (a) the volume of the solid generated by rotating the region around the  $x$ -axis. **DO NOT EVALUATE.**
- (4) (b) the volume of the solid generated by rotating the region around the line  $y = -1$ . **DO NOT EVALUATE.**
- (4) (c) the volume of the solid generated by rotating the region around the line  $x = 3$ . **DO NOT EVALUATE.**
- (4) 7. Consider the integral  $\int_{-1}^2 (1 - x^2)dx$ . Use either the trapezoid rule **OR** Simpson's rule with  $N = 6$  to approximate this integral. **DO NOT EVALUATE.**
- (4) 8. (a) Find the Taylor polynomial of degree 2 for  $f(x) = \sqrt{1+x}$  expanded about  $x = 0$ .
- (3) (b) Use part (a) to approximate  $\sqrt{1.1}$
9. Find the solution of each differential equation:
- (4) (a)  $\frac{dy}{dx} = 3x^2(y^2 + 1); \quad y(0) = 0$
- (4) (b)  $\frac{dy}{dx} + y = 2e^{-x}$
- (4) (c)  $y'' - 2y = 0; \quad y(0) = 0, \quad y'(0) = 1$
- (4) (d)  $2y'' - y' + y = 0$
10. **DO 3 OF THE 5** following problems. Indicate which problem you **DO NOT** want marked. Each is worth 5 marks. You may use the remaining pages in this paper.
- (5) (a) The base of a solid is a semicircle of radius 1. Cross-sections by planes perpendicular to the diameter of the semicircle are squares. Find the volume of the solid.
- (5) (b) A tank contains 1000 litres of brine, consisting of 10 kg salt and water. Pure water enters the tank at a rate of 5  $\ell$ /min. The well mixed solution drains from the tank, also at 5  $\ell$ /min.. At what time does the tank contain exactly 5 kg of salt?
- (5) (c) A spring connected to mass of 1 kg requires 2N of force to compress it 0.2 m from its natural length. The coefficient of friction is  $c = 2$  kg/sec. Find the position of the mass at time  $t$  if the initial displacement is 0 and it is given an initial velocity of 0.1 m/sec in the stretched direction.
- (5) (d) A tank is in the shape of cylinder of radius 3 m and height 10 m. It is filled only to the 9 m level. Find the work required to empty the tank by pumping all of the water through the top of the tank. The density of water is 1000 kg/m<sup>3</sup>. **Do not** assign a value to the gravitational constant  $g$ .
- (5) (e) Integrate  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^x \cos x dx$