

DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 1013

FINAL EXAM

NAME.....

APRIL 20, 2005

STUDENT I.D......

INSTRUCTOR (Please Circle):

(1B) J. Gegenberg

(2B) B. McKellar

(3B) D. Tingley

(4B) D. Kucerovsky

(5B) S. Gupta

(6B) J. Gegenberg

SHOW YOUR WORK. USE THE BACKS OF THE PAGES IF NECESSARY.
NO CALCULATORS, NOTES, TEXTS ALLOWED.

MARKS

- (4) 1. Use the definition of the definite integral as the limit of a Riemann sum to compute $\int_{-1}^3 4x dx$. **Do not** use the Fundamental Theorem of calculus.

2. Evaluate each integral:

(3) (a) $\int \tan x dx$

(3) (b) $\int_0^{\frac{\pi}{3}} \frac{\sin x}{1 + \cos x} dx$

(3) (c) $\int \frac{x^2}{x + 3} dx$

(4) (d) $\int x e^{-x} dx$

(4) (e) $\int \sec^4 t dt$

(5) (f) $\int \frac{3x^2 + 5x - 4}{(x - 1)(x + 1)^2} dx$

(4) (g) $\int \frac{1}{(16 - x^2)^{\frac{3}{2}}} dx$

3. Determine if the improper integrals converge or diverge. If it converges, determine its value.

(4) (a) $\int_0^2 \frac{1}{(x - 1)^2} dx$

(4) (b) $\int_0^{\infty} x e^{-x^2} dx$

(4) 4. Find the total area between $y = \sin x$ and the x -axis from $x = 0$ to $x = \frac{3\pi}{2}$

(4) 5. Set up, **but do not evaluate**, an integral for the arc-length of the curve $y = \ln x$ from $x = 0$ to $x = 1$.

6. Consider the region bounded by the parabola $y = 4x - x^2$ and the line $y = x$. Set up, but do not evaluate, an integral for
- (4) (a) the volume of the solid generated by rotating the region around the x -axis. **DO NOT EVALUATE.**
- (4) (b) the volume of the solid generated by rotating the region around the line $y = -1$. **DO NOT EVALUATE.**
- (4) (c) the volume of the solid generated by rotating the region around the line $x = 3$. **DO NOT EVALUATE.**
- (4) 7. Consider the integral $\int_{-1}^2 (1 - x^2)dx$. Use either the trapezoid rule **OR** Simpson's rule with $N = 6$ to approximate this integral. **DO NOT EVALUATE.**
- (4) 8. (a) Find the Taylor polynomial of degree 2 for $f(x) = \sqrt{1+x}$ expanded about $x = 0$.
- (3) (b) Use part (a) to approximate $\sqrt{1.1}$
9. Find the solution of each differential equation:
- (4) (a) $\frac{dy}{dx} = 3x^2(y^2 + 1); \quad y(0) = 0$
- (4) (b) $\frac{dy}{dx} + y = 2e^{-x}$
- (4) (c) $y'' - 2y = 0; \quad y(0) = 0, \quad y'(0) = 1$
- (4) (d) $2y'' - y' + y = 0$
10. **DO 3 OF THE 5** following problems. Indicate which problem you **DO NOT** want marked. Each is worth 5 marks. You may use the remaining pages in this paper.
- (5) (a) The base of a solid is a semicircle of radius 1. Cross-sections by planes perpendicular to the diameter of the semicircle are squares. Find the volume of the solid.
- (5) (b) A tank contains 1000 litres of brine, consisting of 10 kg salt and water. Pure water enters the tank at a rate of 5 ℓ /min. The well mixed solution drains from the tank, also at 5 ℓ /min.. At what time does the tank contain exactly 5 kg of salt?
- (5) (c) A spring connected to mass of 1 kg requires 2N of force to compress it 0.2 m from its natural length. The coefficient of friction is $c = 2$ kg/sec. Find the position of the mass at time t if the initial displacement is 0 and it is given an initial velocity of 0.1 m/sec in the stretched direction.
- (5) (d) A tank is in the shape of cylinder of radius 3 m and height 10 m. It is filled only to the 9 m level. Find the work required to empty the tank by pumping all of the water through the top of the tank. The density of water is 1000 kg/m³. **Do not** assign a value to the gravitational constant g .
- (5) (e) Integrate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^x \cos x dx$