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DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1503

FINAL EXAMINATION
DECEMBER 2004

TIME: 3 HOURS
TOTAL POINTS = 60

Calculators not permitted. Do each question.

MARKS

1. Consider the two planes $x + y + z = 2$ and $2x + 3y - 5z = 5$ in \mathbb{R}^3 . Find

(2) (a) the angle between these planes;

(4) (b) parametric equations for the line of intersection of the two planes:

$$\begin{cases} x = \\ y = \\ z = \end{cases}$$

(2) 2. Find an equation of the plane which contains the point $P = (5, -6, 7)$ and has normal vector $\vec{n} = (4, 1, -2)$.

3. Consider the following matrix A and vector \vec{b} :

$$A = \begin{bmatrix} 2 & 6 & -4 \\ -2 & -5 & 4 \\ 4 & 11 & -8 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

(4) (a) By using row reduction, reduce the augmented matrix $[A \vec{b}]$.

(1) (b) Using part (a) of this question, explain why A and $[A \vec{b}]$ have the same rank.

(3) (c) Find in vector form the general (or complete) solution to $A\vec{x} = \vec{b}$.

4. Using suitable row operations, the matrix

$$A = \begin{bmatrix} 2 & 1 & -2 & -2 \\ 1 & 0 & -1 & 0 \\ -1 & 1 & 1 & -2 \end{bmatrix}$$

can be reduced to this matrix:

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

(Thus you need not do the reduction here.)

- (3) (a) Find a basis for the nullspace of the original matrix A .
- (2) (b) Find a basis for the column space of the original matrix A .
- (3) (c) Express $\vec{b} = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$ as a linear combination of the basis vectors for the column space found in part (b).
- (4) 5. (a) Find a basis for the subspace of \mathbb{R}^4 spanned by

$$\vec{u} = \begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 4 \\ -7 \\ 2 \\ 5 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 7 \\ 4 \\ -9 \\ -5 \end{bmatrix}.$$

- (1) (b) Using part (a), determine whether the vectors \vec{u} , \vec{v} , \vec{w} are linearly dependent.
- (4) 6. Compute the determinant of the matrix

$$B = \begin{bmatrix} 2 & -3 & 7 \\ -4 & 5 & -14 \\ 8 & -9 & 21 \end{bmatrix},$$

7. In this question, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$.

- (4) (a) Find A^{-1} and verify that $A^{-1}A = I$;
- (4) (b) Factor A into $A = LU$ where L is lower triangular and U is upper triangular.
- (3) (c) Still with A and $\vec{b} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$, solve $A\vec{x} = \vec{b}$ for \vec{x} , by any method you choose.

- (4) 8. (a) Find the eigenvalues and corresponding eigenvectors of $A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$.
- (2) (b) Determine which of the following vectors

$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 2 \end{bmatrix}$$

is an eigenvector of the matrix

$$w = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

and find the corresponding eigenvalue.

- (4) 9. (a) Consider the complex numbers:

$$z = 1 - 3i \quad \text{and} \quad w = 1 + i.$$

Sketch (in the same diagram)

$$z, w, zw \quad \text{and} \quad \bar{z};$$

- (1) (b) (i) Find the polar form $re^{i\theta}$ for $-1 + i$.
- (3) (ii) Find the three complex cube roots of $-1 + i$. (This means solve $u^3 = -1 + i$ in polar form.)
- (2) (c) Find the eigenvalues of $A = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$.