University of New Brunswick — Department of Mathematics & Statistics

Math 1503 — Final Exam — December 11, 2009

Total points: 100 Name: ______________________

Total pages: 8 Student Number: ______________________

Please circle your instructor's name below.

MONSON SALMANI SANKEY TINGLEY

Instructions:

- Calculators and **all** other electronic devices are prohibited.
- Write your solutions in the spaces provided. If you need more room, please use the back of the page and indicate clearly where to find your solution.
- Show your steps and calculations, so that your answers are justified.

1. [20 points] Answer (a)–(j) using a short calculation or explanation.

(a) Find a vector perpendicular to $\mathbf{v} = \langle 1, 2, 3 \rangle$.

(b) Find a unit vector in the direction of $\mathbf{u} = \langle 5, \sqrt{2}, -3 \rangle$.

(c) Find the polar form $re^{i\theta}$ for $-\sqrt{3} + i$.

(d) The change of basis matrix from the standard basis $\mathcal{E} = \{e_1, e_2\}$ to a new basis $\mathcal{B}$ is

$$P_{\mathcal{B}\leftarrow\mathcal{E}} = \begin{bmatrix} 3 & 4 \\ -2 & 9 \end{bmatrix}.$$

Find the coordinate vector of $\mathbf{u} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$ in the basis $\mathcal{B}$.

(e) Does the plane $4x - 3y + 2z = 17$ pass through the origin?

(f) Find a vector **not** in $\text{Span}(\langle 1, -1, 0\rangle, \langle 0, 0, 1\rangle)$.

(g) Find the inverse of $\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$.

(h) One eigenvector of $\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$ is $\mathbf{v} = \begin{bmatrix} -5 \\ 5 \end{bmatrix}$. Find the corresponding eigenvalue.

(i) Find a vector perpendicular to both $\mathbf{u} = \langle 1, 0, 5\rangle$ and $\mathbf{v} = \langle 9, 1, 4\rangle$.

(j) Find the determinant of A.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 7 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. [10 points]

(a) Find the distance between the point $P = (6, 7, 1)$ and the plane $3x - 2y - z = -8$.

(b) Find the equation of **any** line through P that is parallel to the the plane in (a).

3. [12 points] Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} -4 \\ 5 \\ -1 \\ 1 \end{bmatrix}$.

(a) Are $\mathbf{v}_1, \mathbf{v}_2$, and $\mathbf{v}_3$ linearly independent? If not, express one of these vectors as a linear combination of the others.

Question 3 is continued on the next page. →

(b) (Continued from question 3 on the previous page.) Is $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a basis for $\mathbb{R}^4$? Explain why or why not.

4. [10 points] Consider the system

$$\begin{cases} 2x_1 - 4x_2 + x_3 + x_4 = 3 \\ x_1 - 2x_2 + x_4 = 4 \end{cases}$$

(a) Give the augmented matrix M and reduce it.

(b) Clearly describe in column vector form the general solution to the system.

(c) The set of all such solutions $\mathbf{x}$ describes a geometrical object in $\mathbb{R}^4$. What word best describes this object: point, line, plane, other?

5. [12 points] For this question, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 5 \\ 13 \\ 9 \end{bmatrix}$.

(a) Find A^{-1}.

(b) Use A^{-1} to solve $A\mathbf{x} = \mathbf{b}$.

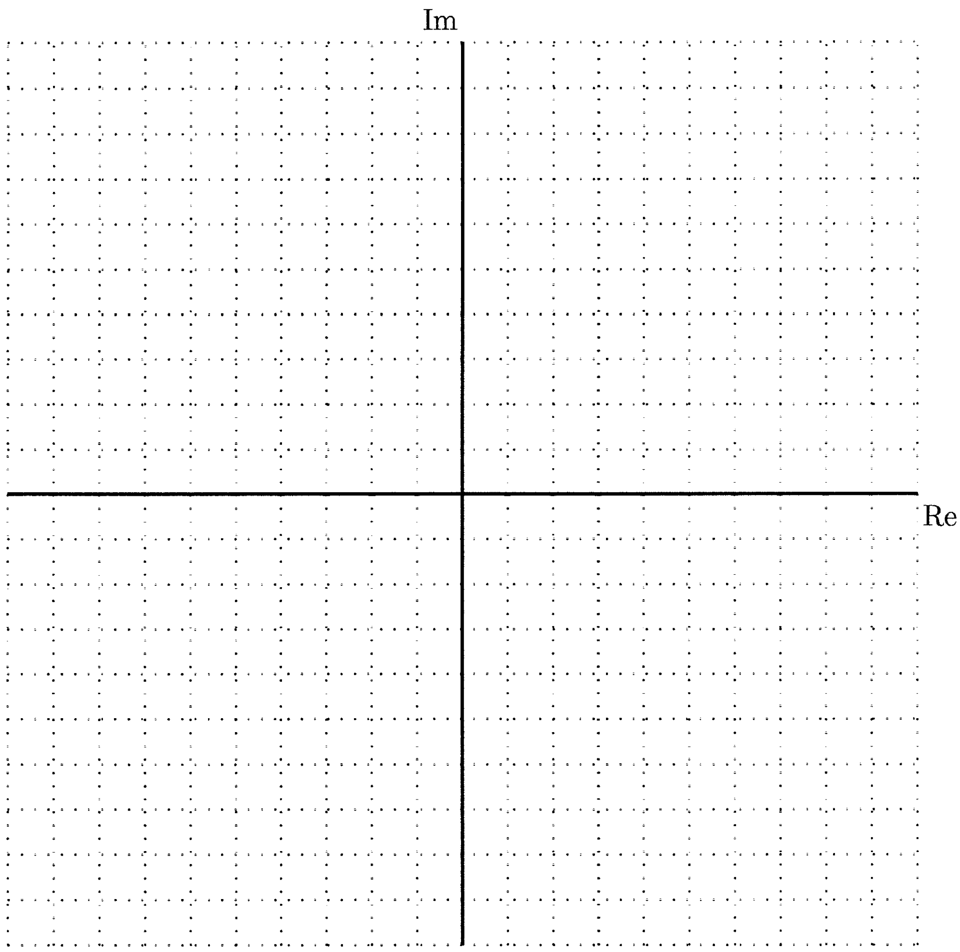
6. [12 points] Find the eigenvalues of

$$B = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 4 & 7 \\ -1 & 0 & -3 \end{bmatrix}$$

and give a basis for each eigenspace. Hint: One of the eigenvalues is 4.

7. [12 points]

(a) Consider the complex number $z = \sqrt{2} + \sqrt{2}i$. Sketch (on the grid provided) z, z^2, z^3, and z^4.



(b) Find all (complex) solutions to $w^4 + 16 = 0$.

8. [12 points]

(a) Find the point of intersection of the plane $x - 2y - z = 21$ and the line $\mathbf{x} = \langle 1, 2, 3\rangle + t\langle 2, -1, 1\rangle$.

(b) Find the angle between the line and the plane in part (a). Indicate this angle in a simple sketch.