

Total points: 75

Name: \_\_\_\_\_

Total pages: 8

Student Number: \_\_\_\_\_

Please circle your instructor's name below.

RANGIPOUR

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**Instructions:**

- Calculators and **all** other electronic devices are prohibited.
- Write your solutions in the spaces provided. If you need more room, please use the back of the page and indicate clearly where to find your solution.
- Unless otherwise indicated, show your steps and/or calculations, and explain your reasoning so that your answers are justified.

Question	Points	Mark
1.1	3	
1.2	3	
1.3	3	
1.4	4	
2.1	5	
2.2	5	
2.3	5	
2.4	10	
2.5	10	
2.6	10	
3.1	10	
3.2	7	

**Part I: On this part of the examination, only your answers will be marked. It is not necessary to justify your answers.**

1. Find the projection of  $\langle 3, 4 \rangle$  onto  $\mathbf{e}_2 = \langle 0, 1 \rangle$ .

2. Find a direction vector for the line given by  $3x - 8y = 11$ .

3. The matrix below is the reduced augmented matrix for a system of linear equations in variables  $x_1, x_2, x_3$ . Write the solution set.

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 2 \end{array} \right]$$

4. Find the eigenvalues of the matrix  $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

**Part II: For this part of the exam, show your work clearly to demonstrate your knowledge of the methods learned in Math 1503.**

1. Find a unit vector perpendicular to both  $\mathbf{u} = \langle 3, -1, 0 \rangle$  and  $\mathbf{v} = \langle -5, 4, 2 \rangle$ .

2.  $\mathcal{C} = \{ \langle 1, -2 \rangle, \langle -4, 7 \rangle \}$  is a basis for  $\mathbb{R}^2$ .

(a) Find the change-of-basis matrix  $P_{\mathcal{C} \leftarrow \mathcal{E}}$  where  $\mathcal{E}$  is the standard basis.

(b) Find the coordinates of  $5\mathbf{i} + 3\mathbf{j}$  relative to  $\mathcal{C}$ .

3. Show that the complex number  $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$  is a 12<sup>th</sup> root of unity. That is, show that  $z^{12} = 1$ .

4. Find the inverse of the matrix  $A$  and verify that your inverse is correct.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

5. (a) Show that there is no solution to the system of equations with augmented matrix

$$\left[ \begin{array}{cc|c} -1 & 1 & 1 \\ 7 & 3 & 5 \\ 3 & 2 & 1 \end{array} \right].$$

(b) Is  $\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$  in the span of  $\begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ ?

(c) Is  $\begin{bmatrix} -3 \\ 11 \\ 4 \end{bmatrix}$  in the span of  $\begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$  and  $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ ?

6. Show that 1 is an eigenvalue for the matrix  $B$  below, and find a basis for the corresponding eigenspace.

$$B = \begin{bmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

**Part III:** For this part of the exam, explain your reasoning carefully to demonstrate your understanding of the concepts you learned in Math 1503.

1. Find the general form ( $ax + by + cz = d$ ) of the equation of the plane containing the point  $P = (5, 4, 3)$  and the line  $L(t) = (1, -4t, 1 + 7t)$ .

2. This question concerns linear transformations from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ .

- (a) Let  $T$  be the linear transformation with matrix  $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  (relative to the standard basis). Explain geometrically what  $T$  does.

- (b) Let  $S$  be the linear transformation that does reflection in the  $x$ -axis. Find the matrix  $B$  of  $S$  (relative to the standard basis).

- (c) Show that the matrices  $A$  and  $B$  from parts (a) and (b) do not commute. That is, show  $AB \neq BA$ .