University of New Brunswick **Department of Mathematics & Statistics**

Math 1503 **Final Exam** **December 16, 2010**

Total points: 75 Name: ____________________

Total pages: 8 Student Number: ____________________

Please circle your instructor's name below.

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Instructions:

- Calculators and **all** other electronic devices are prohibited.
- Write your solutions in the spaces provided. If you need more room, please use the back of the page and indicate clearly where to find your solution.
- Unless otherwise indicated, show your steps and/or calculations, and explain your reasoning so that your answers are justified.

Question	**Points**	**Mark**
1.1	3	
1.2	3	
1.3	3	
1.4	4	
2.1	5	
2.2	5	
2.3	5	
2.4	10	
2.5	10	
2.6	10	
3.1	10	
3.2	7	

Part I: On this part of the examination, only your answers will be marked. It is not necessary to justify your answers.

1. Find the projection of $\langle 3, 4\rangle$ onto $\mathbf{e}_2 = \langle 0, 1\rangle$.

2. Find a direction vector for the line given by $3x - 8y = 11$.

3. The matrix below is the reduced augmented matrix for a system of linear equations in variables x_1, x_2, x_3. Write the solution set.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 1 & 2 \end{array}\right]$$

4. Find the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

Part II: For this part of the exam, show your work clearly to demonstrate your knowledge of the methods learned in Math 1503.

1. Find a unit vector perpendicular to both $\mathbf{u} = \langle 3, -1, 0 \rangle$ and $\mathbf{v} = \langle -5, 4, 2 \rangle$.

2. $\mathcal{C} = \{ \langle 1, -2 \rangle, \langle -4, 7 \rangle \}$ is a basis for $\mathbb{R}^2$.

   (a) Find the change-of-basis matrix $P_{\mathcal{C} \leftarrow \mathcal{E}}$ where $\mathcal{E}$ is the standard basis.

   (b) Find the coordinates of $5\mathbf{i} + 3\mathbf{j}$ relative to $\mathcal{C}$.

3. Show that the complex number $z = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ is a 12^{th} root of unity. That is, show that $z^{12} = 1$.

4. Find the inverse of the matrix A and verify that your inverse is correct.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

5. (a) Show that there is no solution to the system of equations with augmented matrix

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 7 & 3 & 5 \\ 3 & 2 & 1 \end{array}\right].$$

(b) Is $\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$ in the span of $\begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$?

(c) Is $\begin{bmatrix} -3 \\ 11 \\ 4 \end{bmatrix}$ in the span of $\begin{bmatrix} -1 \\ 7 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$?

6. Show that 1 is an eigenvalue for the matrix B below, and find a basis for the corresponding eigenspace.

$$B = \begin{bmatrix} 2 & 1 & -2 \\ 2 & 3 & -4 \\ 1 & 1 & -1 \end{bmatrix}$$

Part III: For this part of the exam, explain your reasoning carefully to demonstrate your understanding of the concepts you learned in Math 1503.

1. Find the general form ($ax+by+cz=d$) of the equation of the plane containing the point $P=(5,4,3)$ and the line $L(t)=(1,-4t,1+7t)$.

2. This question concerns linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.

(a) Let T be the linear transformation with matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (relative to the standard basis). Explain geometrically what T does.

(b) Let S be the linear transformation that does reflection in the x-axis. Find the matrix B of S (relative to the standard basis).

(c) Show that the matrices A and B from parts (a) and (b) do not commute. That is, show $AB \neq BA$.