

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAMINATION

APRIL 17, 2000

TIME: 3 HOURS

NO CALCULATORS

1. (10 marks) Compute each of the following limits or show that it does not exist.

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 4}$

(b) $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^3 + x^2 + 2}$

(c) $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$

(d) $\lim_{x \rightarrow 0} \frac{1}{x^3}$

2. (10 marks)

- (a) Compute the derivative of $y = x^2 + 1$ **using the definition of derivative**.
- (b) Find an equation of the tangent line to the graph of $y = x^2 + 1$ at the point where $x = 1$.

3. (10 marks) Compute the derivative of each of the following functions. **DO NOT SIMPLIFY.**

(a) $y = \frac{1}{x} - 5x^3 + \sqrt{x}$

(b) $y = e^{-x^2} + 2$

(c) $y = \ln(x^2 + x)$

(d) $y = \frac{x}{\sqrt{x^2 + 1}}$

4. (15 marks) Let $f(x) = \frac{x}{x^2 + 1}$. You may take for granted that $f'(x) = \frac{1 - x^2}{(x^2 + 1)^2}$ and $f''(x) = \frac{2x(x^2 - 3)}{(x^2 + 1)^3}$.

- (a) Find the critical points.
- (b) Use the **First Derivative Test** to determine whether the **negative** critical point is a local maximum, local minimum, or neither.
- (c) Use the **Second Derivative Test** to determine whether the **positive** critical point is a local maximum, local minimum, or neither.
- (d) Find the absolute maximum of $f(x)$ on the interval where $0 \leq x \leq 2$.

5. (10 marks) Let $f(x) = 3x - x^3$.
- (a) Determine the intervals where $f(x)$ is increasing and the intervals where it is decreasing.
 - (b) Determine the intervals where $f(x)$ is concave up and the intervals where it is concave down.
6. (15 marks) Weasel Inc., a publisher of calculus textbooks, knows that $5000 - 50x$ books will be sold if the price is x dollars per book. The cost to Weasel Inc. is a one time fee of \$ 5000, plus \$ 10 per each sold book (shipping and handling).
- (a) Express Weasel's revenue as a function $R(x)$.
 - (b) Express Weasel's costs as a function $C(x)$.
 - (c) Using the formula for profit, $P(x) = R(x) - C(x)$, find the value of x so that Weasel Inc. makes the maximum possible profit.
7. (10 marks) Evaluate each of the following integrals:
- (a) $\int (x^2 - \sqrt{x} + \frac{1}{x}) dx$
 - (b) $\int x^2 \sqrt{x^3 + 1} dx$
 - (c) $\int (e^{2x} + 1) dx$
 - (d) $\int_0^1 \frac{1}{(x+1)^2} dx$
8. (10 marks) Compute the area of the region between the graph of $y = 1 - x^2$ and that part of the x -axis where $0 \leq x \leq 2$.