Department of Mathematics and Statistics
University of New Brunswick, Fredericton
Math 1823 Final Exam — Winter 2001
April 24, 2001

CALCULATORS NOT PERMITTED.
SHOW ALL DETAILS OF YOUR WORK.

1. (10 marks) Compute each of the following limits or show that it does not exist:

   (a) $\lim\limits_{x\to 0} \dfrac{x^2+1}{x-1}$

   (b) $\lim\limits_{x\to 1} \dfrac{x^2-3x+2}{x^2-4x+3}$

   (c) $\lim\limits_{x\to 5} \dfrac{1}{(x-5)^3}$

   (d) $\lim\limits_{x\to\infty} \dfrac{x^3+3x+1}{2x^3-3}$

2. (10 marks)

   (a) Compute the derivative of $f(x)=\sqrt{x}$ **using the definition of derivative.**

   (b) Find the equation of the tangent line to the graph of $y=\sqrt{x}$ at the point where $x=4$.

3. (20 marks) Compute the derivative of each the following functions (**DO NOT SIMPLIFY!**):

   (a) $2x^6 - \dfrac{3}{x} + \dfrac{1}{\sqrt{x}}$

   (b) $x^2e^x + \dfrac{\ln x}{x}$

   (c) $\sqrt{\dfrac{x}{x+1}}$

(d) $\ln(e^{x^2} + 1)$

4. (10 marks) A function $f(x)$ is such that $f'(x) = 6x^2 - 6x - 12$.

   (a) Determine the intervals where $f(x)$ is increasing and intervals where it is decreasing.

   (b) Determine the intervals where $f(x)$ is concave up and intervals where it is concave down.

   (c) Use the **First Derivative Test** to determine whether the critical point $x = -1$ is a local minimum or a local maximum.

   (d) Use the **Second Derivative Test** to determine whether the critical point $x = 2$ is a local minimum or a local maximum.

5. (10 marks) A manufacturer whose production cannot exceed 8 units per day has cost and revenue functions given by

$$C(x) = 2x^3 + x^2 - 100x + 200$$

$$R(x) = x^2 + 50x$$

   where x denotes the number of units produced daily. How many units should be produced daily to make sure that the profit is maximal? (Profit is given by the formula $P(x) = R(x) - C(x)$.)

6. (10 marks) A manufacturer of bicycles finds that when x bicycles are produced, the following costs are incurred: a fixed cost of \$ 1000, labor cost of \$ 10 per bicycle, and a cost of $\frac{25000}{x}$ dollars for advertizing. How many bicycles should be produced to minimize the total cost?

7. (20 marks) Compute each of the following integrals:

   (a) $\int (\frac{1}{2x} + 3e^x + \frac{1}{\sqrt{x}})dx$

   (b) $\int \frac{1 + x^{1/3}}{\sqrt{x}} dx$

   (c) $\int \frac{5x}{x^2 + 1} dx$

(d) $\displaystyle\int_1^e \frac{(\ln x)^2}{x} dx$

(e) $\displaystyle\int_0^2 \frac{dx}{x+1}$

8. (10 marks) Find the area between the graph of $y = 4 - x^2$ and the x-axis from $x = -2$ to $x = 3$.