

Department of Mathematics and Statistics  
University of New Brunswick, Fredericton  
Math 1823 Final Exam — Winter 2001  
April 24, 2001

**CALCULATORS NOT PERMITTED.  
SHOW ALL DETAILS OF YOUR WORK.**

1. (10 marks) Compute each of the following limits or show that it does not exist:

(a)  $\lim_{x \rightarrow 0} \frac{x^2 + 1}{x - 1}$

(b)  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

(c)  $\lim_{x \rightarrow 5} \frac{1}{(x - 5)^3}$

(d)  $\lim_{x \rightarrow \infty} \frac{x^3 + 3x + 1}{2x^3 - 3}$

2. (10 marks)

- (a) Compute the derivative of  $f(x) = \sqrt{x}$  using the definition of derivative.
- (b) Find the equation of the tangent line to the graph of  $y = \sqrt{x}$  at the point where  $x = 4$ .

3. (20 marks) Compute the derivative of each the following functions (**DO NOT SIMPLIFY!**):

(a)  $2x^6 - \frac{3}{x} + \frac{1}{\sqrt{x}}$

(b)  $x^2 e^x + \frac{\ln x}{x}$

(c)  $\sqrt{\frac{x}{x+1}}$

(d)  $\ln(e^{x^2} + 1)$

4. (10 marks) A function  $f(x)$  is such that  $f'(x) = 6x^2 - 6x - 12$ .
- (a) Determine the intervals where  $f(x)$  is increasing and intervals where it is decreasing.
  - (b) Determine the intervals where  $f(x)$  is concave up and intervals where it is concave down.
  - (c) Use the **First Derivative Test** to determine whether the critical point  $x = -1$  is a local minimum or a local maximum.
  - (d) Use the **Second Derivative Test** to determine whether the critical point  $x = 2$  is a local minimum or a local maximum.
5. (10 marks) A manufacturer whose production cannot exceed 8 units per day has cost and revenue functions given by

$$C(x) = 2x^3 + x^2 - 100x + 200$$

$$R(x) = x^2 + 50x$$

where  $x$  denotes the number of units produced daily. How many units should be produced daily to make sure that the profit is maximal? (Profit is given by the formula  $P(x) = R(x) - C(x)$ .)

6. (10 marks) A manufacturer of bicycles finds that when  $x$  bicycles are produced, the following costs are incurred: a fixed cost of \$ 1000, labor cost of \$ 10 per bicycle, and a cost of  $\frac{25000}{x}$  dollars for advertizing. How many bicycles should be produced to minimize the total cost?
7. (20 marks) Compute each of the following integrals:

(a)  $\int \left( \frac{1}{2x} + 3e^x + \frac{1}{\sqrt{x}} \right) dx$

(b)  $\int \frac{1 + x^{1/3}}{\sqrt{x}} dx$

(c)  $\int \frac{5x}{x^2 + 1} dx$

(d)  $\int_1^e \frac{(\ln x)^2}{x} dx$

(e)  $\int_0^2 \frac{dx}{x+1}$

8. (10 marks) Find the area between the graph of  $y = 4 - x^2$  and the  $x$ -axis from  $x = -2$  to  $x = 3$ .