

**DEPARTMENT OF MATHEMATICS & STATISTICS**  
**MATH 1823**  
**FINAL EXAMINATION**

DECEMBER 2001

TIME: 3 hours

**INSTRUCTIONS:** Answer all questions, and show the details of your solutions.

MARKS

1. Differentiate the following functions with respect to  $x$ . **DO NOT SIMPLIFY.**

(5) (a)  $y = 3x^3 - \frac{2}{x^2} - \frac{1}{2\sqrt{x}}$

(5) (b)  $y = (x^3 + 3x^2)^4$

(5) (c)  $y = (3x + 2)\sqrt{x^2 + 4}$

(5) (d)  $y = e^{3x} + e^{x^2}$

(5) (e)  $y = \ln(2x) + \frac{1}{\ln x}$

(9) 2. If  $y = \frac{3x^2 + 4x}{3x^2 + 4x + 1}$  find  $y'$ , simplify it, and find the equation of the tangent line at the point where  $x = 1$ .

(9) 3. Use the definition of  $f'(x)$  as a limit to find  $f'(2)$  when  $f(x) = \frac{1}{x}$ .

(6) 4. Consider the function

$$f(x) = \begin{cases} x^2 & -1 \leq x < 1 \\ -x + 2 & 1 \leq x < 3 \\ 2x - 6 & x \geq 3 \end{cases}$$

Is  $f(x)$  continuous at  $x = 1$ ? at  $x = 3$ ? (Give reasons.)

(9) 5. Fredericton Transit sells 2900 bus passes each month at \$25 each. Market research indicates that for every dollar increase, they will sell 100 fewer passes. What price will maximize the revenue?

(9) 6. A publishing company sells 400,000 copies of a dictionary every year. Each production run costs \$1,000, and storage costs are \$0.50 per book based on the average number in stock. How many dictionaries should be printed in each run to minimize costs?

(9) 7. Use the first and second derivatives to determine all the extreme points of  $y = x^4 - 8x^2 + 1$ .

8. Evaluate

(6) (a)  $\int \frac{1}{x^2} - \frac{2}{x} + \frac{1}{3x^{1/2}} dx$

(6) (b)  $\int_0^1 3x^{3/2} - e^{3x} + 7 dx$

(5) 9. Which of the following is a correct value of  $\int x e^x dx$  (Give reasons.)

(a)  $e^{x^2}$

(b)  $x e^x - e^x$

(c)  $\frac{x^2}{2} e^x$

(7) 10. A plant producing  $x$  units has marginal cost  $MC(x) = .1x^2 - x + 200$  and marginal revenue  $MR(x) = 300 + 49x + .4x^2$ . Find the extra profit if production is increased from 10 to 20 units.