

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAMINATION

APRIL, 2002

TIME: 3 hours

MARKS

1. For the function:

$$f(x) = \begin{cases} x & , 0 \leq x \leq 4 \\ \frac{1}{2}x + 2 & , 4 < x \leq 5 \\ 5 & , 5 < x < 7 \end{cases}$$

- (4) (a) Graph it.

- (1) (b) Where is  $f(x)$  continuous?

- (1) (c) Where is  $f(x)$  differentiable?

- (4) 2. Calculate the following limits:

(a)  $\lim_{x \rightarrow -3} \frac{x^2 + 3x}{x + 3}$

(b)  $\lim_{x \rightarrow \infty} \frac{4x^2 - 1}{7 - 2x + 8x^2}$

- (8) 3. Use the definition of the derivative as limit to find the derivative of  $f(x) = \sqrt{3x - 2}$ . Check your answer using the rules of differentiation.

4. Find the derivatives of the following functions. (**Do not simplify!**)

(3) (a)  $f(x) = \frac{100}{x^3} - \frac{50}{x^{1/2}} + 10x - 1$

(4) (b)  $g(x) = \frac{4x^2 + 3}{2x - 1}$

(4) (c)  $h(x) = 3x\sqrt{4x^2 - 5}$

(4) (d)  $f(x) = \ln(x^3) - (\ln x)^3$

(3) (e)  $g(x) = e^{-4x^2 + 3x - 8}$

- (9) 5. Use the first and second derivative to sketch the graph of

$$y = 3x^4 - 4x^3 + 1$$

- (8) 6. For a product the fixed costs are \$1200 and the variable costs are \$2 per unit. The demand is given by  $p = \frac{100}{\sqrt{x}}$

- (a) Find the revenue.

- (b) Find the profit.  
(c) Find the value of  $x$  that will maximize the profit. Justify your answer.

(2) 7. Solve for  $x$  :  $e^{2x-5} + 1 = 4$ .

8. Find the following integrals:

(3) (a)  $\int (3t^4 - 4t + 5) dt$

(4) (b)  $\int \frac{(2x^3 + 3x)}{(x^4 + 3x^2 + 7)^4} dx$

(4) (c)  $\int \left( \frac{4}{(x-1)^2} + \frac{4}{x-1} \right) dx$

(4) 9. If the marginal cost is

$$.0003x^2 - .04x + 5$$

find the cost function given that the fixed cost is \$500.

(5) 10. Find the area between the curves

and  $y = 8 + 4x - x^2$

$$y = x^2 - 2x.$$