

**Department of Mathematics and Statistics**  
**University of New Brunswick, Fredericton**  
**Math 1823 Final Examination — April 2003**  
**Time: 3 hours**  
**JUSTIFY YOUR ANSWERS**

1. (6 marks) Evaluate the following limits:

(a)  $\lim_{x \rightarrow \infty} \frac{3x^3 + x + 1}{-x^3 + x^2 + 7}$

(b)  $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$

(c)  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

2. (6 marks)

(a) Use the definition of derivative as limit to compute  $f'(4)$ , where  $f(x) = \sqrt{x}$ .

(b) Find the equation of the tangent line to the curve  $y = \sqrt{x}$  at the point where  $x = 4$

3. (10 marks) Compute the derivative of each of the following functions:

(a)  $y = \frac{x}{3} + \frac{3}{x}$

(b)  $y = \frac{x^2 - 1}{x^2 + 1}$

(c)  $y = \sqrt{x^3 + 1}$

(d)  $y = \ln(x^3 + 3x + 1)$

(e)  $y = x^2 e^{x^2 - x}$

4. (10 marks) Let  $f(x) = 2x^3 - 15x^2 + 36x + 24$ .

(a) Determine the intervals where  $f$  is increasing and where  $f$  is decreasing.

(b) Determine the intervals where  $f$  is concave up and where  $f$  is concave down.

- (c) Use the **first derivative test** to determine whether  $x = 2$  is a relative minimum or a relative maximum.
- (d) Use the **second derivative test** to determine whether  $x = 3$  is a relative minimum or a relative maximum.
- (e) Sketch the graph of  $f(x)$ .
5. (6 marks) A closed rectangular box with square base and a volume of 12 cubic feet is to be constructed using two different types of materials. The top is made of a metal costing \$2 per square foot and the remainder of wood costing \$1 per square foot. Find the dimension of the box for which the cost of materials is minimized.
6. (6 marks) A fitness club offers memberships at the rate of \$200, provided that a minimum of 100 people join. For each member in excess of 100, the membership fee will be reduced by \$1 per person (for each member). At most 160 memberships can be sold. How many memberships should the club try to sell in order to maximize its revenue?
7. (8 marks) Compute each of the following integrals:
- (a)  $\int \left( \frac{1}{\sqrt{x}} + 3x^3 \right) dx$
- (b)  $\int \frac{x}{e^{x^2}} dx$
- (c)  $\int \ln \sqrt{x} dx$
- (d)  $\int_0^1 \frac{x}{x^2 + 1} dx$
8. (4 marks) Find the area of the region bounded by the curves  $y = x^2$  and  $y = 18 - x^2$ .
9. (4 marks) Find a function  $f(x)$  such that  $f'(x) = x^2 + \sqrt{x}$  and  $f(1) = 3$ .