

DEPARTMENT OF MATHEMATICS & STATISTICS  
MATH 1823

FINAL EXAM  
DECEMBER 2004

TIME: 3 HOURS

NO CALCULATORS. SHOW ALL WORK ON THESE PAGES.

MARKS

1.  $f(x) = \begin{cases} x + 2 & x < 0 \\ x^2 + 2 & 0 \leq x < 2 \\ 5 & x \geq 2 \end{cases}$

- (4) (a) Draw the graph of  $f(x)$ .
- (2) (b) Is  $f(x)$  continuous at  $x = 2$ . Justify your answer.
- (2) (c) Where is  $f(x)$  not differentiable? Justify your answer.

2. Evaluate the following limits:

(3) (a)  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

(3) (b)  $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x^2 - 7}{x^3 + 5x - 3}$

(3) (c)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$

- (4) 3. (a) Using the definition of the derivative, prove that the derivative of  $f(x) = 2x^2 + 5x + 1$  is  $4x + 5$ .
- (3) (b) For  $f(x)$  in part (a), find the equation of the tangent line when  $x = -2$ .
- (3) (c) For  $f(x)$  in part (a), find the point where the tangent line is horizontal.

4. Find the derivative of the following: (**DO NOT SIMPLIFY**)

(4) (a)  $f(x) = 3x^3 + \frac{2}{x} + 2\sqrt{x}$

(4) (b)  $f(x) = (x^2 + 1)^2(x^3 + 3)$

(4) (c)  $f(x) = \ln(x^2 + 1) + 3e^{2x}$

(4) (d)  $f(x) = \frac{(x^2 + 1)^3}{x^3 - 7}$

- (15) 5. (a) Use the first and second derivative rules to sketch the graph of: (**Be sure to show all work clearly**)

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 5.$$

- (1) (b) State intervals of increase/decrease.

- (1) (c) State intervals of concavity.

- (1) (d) State Domain and Range of  $f(x)$ .

6. Find the integrals of the following:

(4) (a)  $\int (2x^3 + 3x^2) dx$

(4) (b)  $\int \left( \frac{2}{x} + e^{2x} + 5 \right) dx$

(4) (c)  $\int_1^4 (2x + 7) dx$

7. For the function  $y = x^2 + 1$  and  $y = 2x + 4$ ,

- (4) (a) sketch their graphs;

- (4) (b) find the area between the functions.

- (5) 8. (a) A storage container is to be built in the shape of a box with a square base. It is to have a volume of 150 cubic feet. The concrete for the base costs \$4.00/sq. ft., the material for the roof costs \$2.00/sq. ft., and the material for the sides costs \$2.50/sq.ft. Find the dimensions to minimize the cost of the container.

- (b) For a certain product, the fixed costs are \$1500 and the variable costs are \$2/unit. The demand equation is given by  $p = \frac{100}{\sqrt{x}}$ , where  $x$  is the quantity,

- (2) (i) find the cost function ( $C(x)$ );

- (2) (ii) find the revenue function ( $R(x)$ );

- (2) (iii) find the profit function ( $P(x)$ );

(4) (iv) find the value of  $x$  that will maximize the profit.

(4) (c) If the marginal cost is given by  $MC(x) = .03x^2 - .02x + 5$ , find the cost function given that the fixed cost is \$500.

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