DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAM
DECEMBER 2004 **TIME: 3 HOURS**

NO CALCULATORS. SHOW All WORK ON THESE PAGES.

MARKS

1. $f(x) = \begin{cases} x+2 & x < 0 \\ x^2+2 & 0 \le x < 2 \\ 5 & x \ge 2 \end{cases}$

(4) (a) Draw the graph of $f(x)$.

(2) (b) Is $f(x)$ continuous at $x = 2$. Justify your answer.

(2) (c) Where is $f(x)$ not differentiable? Justify your answer.

2. Evaluate the following limits:

(3) (a) $\lim_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$

(3) (b) $\lim_{x \to \infty} \dfrac{3x^3 + 2x^2 - 7}{x^3 + 5x - 3}$

(3) (c) $\lim_{x \to 0} \dfrac{\sqrt{x+9} - 3}{x}$

(4) 3. (a) Using the definition of the derivative, prove that the derivative of $f(x) = 2x^2 + 5x + 1$ is $4x + 5$.

(3) (b) For $f(x)$ in part (a), find the equation of the tangent line when $x = -2$.

(3) (c) For $f(x)$ in part (a), find the point where the tangent line is horizontal.

4. Find the derivative of the following: (**DO NOT SIMPLIFY**)

(4) (a) $f(x) = 3x^3 + \dfrac{2}{x} + 2\sqrt{x}$

(4) (b) $f(x) = (x^2+1)^2(x^3+3)$

(4) (c) $f(x) = \ln(x^2+1) + 3e^{2x}$

(4) (d) $f(x) = \dfrac{(x^2+1)^3}{x^3-7}$

(15) 5. (a) Use the <u>first</u> and <u>second</u> derivative rules to sketch the graph of: (**Be sure to show <u>all</u> work clearly**)

$$f(x) = \frac{1}{3}\,x^3 - x^2 - 3x + 5.$$

(1) (b) State intervals of increase/decrease.

(1) (c) State intervals of concavity.

(1) (d) State Domain and Range of $f(x)$.

6. Find the integrals of the following:

(4) (a) $\displaystyle\int (2x^3 + 3x^2)\, dx$

(4) (b) $\displaystyle\int \left(\frac{2}{x} + e^{2x} + 5\right)\, dx$

(4) (c) $\displaystyle\int_1^4 (2x+7)\, dx$

7. For the function $y = x^2 + 1$ and $y = 2x + 4$,

(4) (a) sketch their graphs;

(4) (b) find the area between the functions.

(5) 8. (a) A storage container is to be built in the shape of a box with a square base. It is to have a volume of 150 cubic feet. The concrete for the base costs \$4.00/sq. ft., the material for the roof costs \$2.00/sq. ft., and the material for the sides costs \$2.50/sq.ft. Find the dimensions to minimize the cost of the container.

(b) For a certain product, the fixed costs are \$1500 and the variable costs are \$2/unit. The demand equation is given by $p = \dfrac{100}{\sqrt{x}}$, where x is the quantity,

(2) (i) find the cost function ($C(x)$);

(2) (ii) find the revenue function ($R(x)$);

(2) (iii) find the profit function ($P(x)$);

(4) (iv) find the value of x that will maximize the profit.

(4) (c) If the marginal cost is given by $MC(x) = .03x^2 - .02x + 5$, find the cost function given that the fixed cost is \$500.

(100)