

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAMINATION  
APRIL 2005

TIME: 3 HOURS  
TOTAL POINTS = 85

NO CALCULATORS

MARKS

- (9) 1. Calculate the following limits:

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - 6x + 8}{x - 2}$

(b)  $\lim_{x \rightarrow 4} 9x^2 - \frac{2}{x} - 4$

(c)  $\lim_{x \rightarrow \infty} \frac{3x^3 + 2x^2 + 1}{2x^3 - 5}$

- (5) 2. (a) Use the definition of the derivative to find the derivative of  $f(x) = 3x^2$ .

- (5) (b) Find the equation of a tangent line to the above function when  $x = 2$ .

- (15) 3. Find the derivatives of the following. (**DO NOT SIMPLIFY**)

(a)  $f(x) = \frac{x^2}{2} + \sqrt{x} + \frac{1}{x}$

(b)  $f(x) = (3x^3 + 6x + 7)(x^2 + 5)$

(c)  $f(x) = \frac{(2x^2 - 1)}{(x^2 + 3)}$

(d)  $f(x) = (\ln 3x)(e^{2x})$

(e)  $f(x) = \frac{1}{(2x^3 + 3x + 1)^2}$

- (16) 4. Sketch the graph of

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - 4x + 1.$$

Show all work!

Be sure to include the domain of  $f$ , intervals of increase and decrease, local extrema, concavity, points of inflection and intercepts (if possible).

- (5) 5. A farmer wishes to fence in  $60,000m^2$  of land in a rectangular field along a straight road. The fencing that he plans to use along the road costs  $\$10/m$  and the fencing for the other 3 sides costs  $\$5/m$ . Find the dimensions of the field which minimize his cost.

- (5) 6. Suppose that the demand function for a certain product is  $p = 100 - .01x$  and the cost function is  $c(x) = 50x + 10,000$ . Find the value of  $x$  (quantity) that maximizes the profit. (Prove this is a maximum.)

- (12) 7. Compute the following integrals:

(a)  $\int (x^3 + 3x^2) dx$

(b)  $\int \left( \frac{1}{x} + e^x \right) dx$

(c)  $\int 2x(x^2 + 3)^2 dx$

(d)  $\int_1^3 (5x^2) dx$

- (8) 8. Find the area of the region which lies between the curves  $y = 8 - x^2$  and  $y = x^2$ .

- (5) 9. A company has determined its marginal cost to be

$$C' = x^2 - 3x.$$

Find the cost function given that the fixed cost is \$1000.