DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAMINATION — TIME: 3 HOURS
APRIL 2005 — TOTAL POINTS = 85

NO CALCULATORS

MARKS

(9) 1. Calculate the following limits:

(a) $\displaystyle\lim_{x\to 2} \frac{x^2-6x+8}{x-2}$

(b) $\displaystyle\lim_{x\to 4} 9x^2 - \frac{2}{x} - 4$

(c) $\displaystyle\lim_{x\to \infty} \frac{3x^3+2x^2+1}{2x^3-5}$

(5) 2. (a) Use the definition of the derivative to find the derivative of $f(x) = 3x^2$.

(5) (b) Find the equation of a tangent line to the above function when $x = 2$.

(15) 3. Find the derivatives of the following. (**DO NOT SIMPLIFY**)

(a) $f(x) = \dfrac{x^2}{2} + \sqrt{x} + \dfrac{1}{x}$

(b) $f(x) = (3x^3+6x+7)(x^2+5)$

(c) $f(x) = \dfrac{(2x^2-1)}{(x^2+3)}$

(d) $f(x) = (\ln 3x)(e^{2x})$

(e) $f(x) = \dfrac{1}{(2x^3+3x+1)^2}$

(16) 4. Sketch the graph of

$$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - 4x + 1.$$

Show all work!

Be sure to include the domain of f, intervals of increase and decrease, local extrema, concavity, points of inflection and intercepts (if possible).

(5) 5. A farmer wishes to fence in $60{,}000m^2$ of land in a rectangular field along a straight road. The fencing that he plans to use along the road costs $\$10/m$ and the fencing for the other 3 sides costs $\$5/m$. Find the dimensions of the field which minimize his cost.

(5) 6. Suppose that the demand function for a certain product is $p = 100 - .01x$ and the cost function is $c(x) = 50x + 10,000$. Find the value of x (quantity) that maximizes the profit. (Prove this is a maximum.)

(12) 7. Compute the following integrals:

(a) $\int (x^3 + 3x^2)\, dx$

(b) $\int \left(\frac{1}{x} + e^x\right)\, dx$

(c) $\int 2x(x^2 + 3)^2\, dx$

(d) $\int_1^3 (5x^2)\, dx$

(8) 8. Find the area of the region which lies between the curves $y = 8 - x^2$ and $y = x^2$.

(5) 9. A company has determined its marginal cost to be

$$C' = x^2 - 3x.$$

Find the cost function given that the fixed cost is \$1000.