DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAMINATION — TIME: 3 HOURS

DECEMBER 1999 — TOTAL POINTS = 90

INSTRUCTIONS:

(a) Answer all questions, and show the details of your solutions. Credit will be given to method and presentation of solutions.

(b) **The use of calculators is not permitted.**

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MARKS

(9) 1. Evaluate the following limits:

(a) $\displaystyle\lim_{x\to -2^+} \frac{x^2-4}{x+2}$

(b) $\displaystyle\lim_{x\to -2^-} \frac{x^2+4}{x+2}$

(c) $\displaystyle\lim_{x\to -\infty} \frac{4-x^2}{4+x^2}$

(6) 2. Determine the values of x where the function $f(x) = \dfrac{x^2-1}{\sqrt{x^2+2x-8}}$ is continuous.

(8) 3. Use the definition of the derivative as a limit to find the derivative of the function $f(x) = 6x^2 - 4x + 15$.

(16) 4. Find the derivatives of the following functions:

(a) $f(x) = x^3 - 5\sqrt{x} + \ln(3x)$.

(b) $g(x) = (x^3 - 4x + 1)^{3/2}$

(c) $h(x) = \dfrac{4x^2+3}{2x-1}$

(d) $F(x) = 5xe^{-x^2}$

(6) 5. Find an equation for the tangent line to the curve $y = x^3 - 2x^2 - 2x + 1$ at $x = 2$.

(8) 6. A box with an open top and a square base is to be constructed to have a volume of $32m^3$. What dimensions should the box have to minimize the surface area of the constructive material?

(5) 7. The revenue function for a particular product is $R(x) = 5,000x - x^2$ where x is the number of units produced in a month. In October, the production level was 1,000 units. At what rate is the revenue increasing per month in October if the production is increasing at a rate of 40 units per month?

(12) 8. Compute the following integrals:

(a) $\displaystyle\int \left(x^{3/2} - \frac{4}{x} + \frac{5}{\sqrt{x}} \right) \, dx$

(b) $\displaystyle\int (x^3 + x)^{1/2}(3x^2 + 1) \, dx$

(c) $\displaystyle\int_2^{\infty} \left(\frac{1}{x^2} + e^{-4x} \right) \, dx$

(6) 9. Find the area bounded by the curves $y = x^2 + 1$ and $y = 2x + 1$.

(9) 10. Let $f(x) = -x^3 + 9x^2 - 15x - 5$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Determine the intervals of x over which the function is increasing/decreasing.

(c) Also, determine the intervals over which $f(x)$ is concave up/concave down.

(d) Find and classify the critical points of $f(x)$.

(e) Draw a neat sketch of the graph of $f(x)$.

(5) 11. The marginal cost for a product $MC(x)$ $(= C'(x))$ is given by $MC(x) = 2x^2 + 75$. If the costs are \$22,000 to product 30 units, what is the cost function?