

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1823

FINAL EXAMINATION
DECEMBER 1999

TIME: 3 HOURS
TOTAL POINTS = 90

INSTRUCTIONS:

- (a) Answer all questions, and show the details of your solutions. Credit will be given to method and presentation of solutions.
- (b) **The use of calculators is not permitted.**

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MARKS

- (9) 1. Evaluate the following limits:
 - (a) $\lim_{x \rightarrow -2^+} \frac{x^2 - 4}{x + 2}$
 - (b) $\lim_{x \rightarrow -2^-} \frac{x^2 + 4}{x + 2}$
 - (c) $\lim_{x \rightarrow -\infty} \frac{4 - x^2}{4 + x^2}$
- (6) 2. Determine the values of x where the function $f(x) = \frac{x^2 - 1}{\sqrt{x^2 + 2x - 8}}$ is continuous.
- (8) 3. Use the definition of the derivative as a limit to find the derivative of the function $f(x) = 6x^2 - 4x + 15$.
- (16) 4. Find the derivatives of the following functions:
 - (a) $f(x) = x^3 - 5\sqrt{x} + \ln(3x)$.
 - (b) $g(x) = (x^3 - 4x + 1)^{3/2}$
 - (c) $h(x) = \frac{4x^2 + 3}{2x - 1}$
 - (d) $F(x) = 5xe^{-x^2}$
- (6) 5. Find an equation for the tangent line to the curve $y = x^3 - 2x^2 - 2x + 1$ at $x = 2$.
- (8) 6. A box with an open top and a square base is to be constructed to have a volume of $32m^3$. What dimensions should the box have to minimize the surface area of the constructive material?
- (5) 7. The revenue function for a particular product is $R(x) = 5,000x - x^2$ where x is the number of units produced in a month. In October, the production level was 1,000 units. At what rate is the revenue increasing per month in October if the production is increasing at a rate of 40 units per month?

(12) 8. Compute the following integrals:

(a) $\int \left(x^{3/2} - \frac{4}{x} + \frac{5}{\sqrt{x}} \right) dx$

(b) $\int (x^3 + x)^{1/2} (3x^2 + 1) dx$

(c) $\int_2^\infty \left(\frac{1}{x^2} + e^{-4x} \right) dx$

(6) 9. Find the area bounded by the curves $y = x^2 + 1$ and $y = 2x + 1$.

(9) 10. Let $f(x) = -x^3 + 9x^2 - 15x - 5$.

(a) Compute $f'(x)$ and $f''(x)$.

(b) Determine the intervals of x over which the function is increasing/decreasing.

(c) Also, determine the intervals over which $f(x)$ is concave up/concave down.

(d) Find and classify the critical points of $f(x)$.

(e) Draw a neat sketch of the graph of $f(x)$.

(5) 11. The marginal cost for a product $MC(x)$ ($= C'(x)$) is given by $MC(x) = 2x^2 + 75$. If the costs are \$22,000 to product 30 units, what is the cost function?