

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1833

FINAL EXAMINATION
DECEMBER 2000

TIME: 3 HOURS
TOTAL POINTS = 60

INSTRUCTIONS:

- (a) Show **ALL** intermediate calculations.
- (b) Calculators are permitted.

FORMULAS:

$$A = P(1 + rt)$$

$$A = P(1 + i)^n$$

$$S = Rs_n|i = R \left[\frac{(1 + i)^n - 1}{i} \right]$$

$$P = Ra_n|i = R \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

- (3) 1. A company sells frisbees for \$6 each. Variable costs are \$2 per frisbee. Fixed costs per week are \$37,500. How many frisbees per week must be sold to break even?
- (3) 2. The population of a small city is approximately a linear function of time. In 1990, the population was 15,000. In 1996, it was 20,000. What is its population in 2000?
- (5) 3. If $A = \begin{bmatrix} 3 & -1 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 & 2 \\ 6 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, find the following, if possible:
 - (a) $A + 2B$,
 - (b) $3A - C$,
 - (c) AC ,
 - (d) CA .
- (4) 4. Solve the following system using matrices and row reduction. Show all intermediate steps.

$$\begin{cases} x - y - z = -3 \\ 3x - 4y - 2z = -11 \\ 5x - 6y - 4z = -17 \end{cases}.$$

- (5) 5. Find the inverse of the following matrix, if it exists.

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ -1 & 3 & -4 \end{bmatrix}.$$

- (4) 6. Set up the following as a linear programming problem. Do not solve it!
- A shop sells a mixture of cashews and pecans for a profit of \$3 a pound. \$1,000 has been allocated for buying cashews at \$2 a pound and pecans for \$1.50 a pound. The mixture must contain at least twice as many pecans as cashews (by weight), but no more than 3 times as many (by weight). How many pounds of each type should be ordered to maximize the profit?

- (5) 7. Find the maximum and minimum values of

$$P = 4x + 3y$$

subject to

$$5x + 3y \leq 30$$

$$2x + 3y \leq 21$$

$$x \geq 0$$

$$y \geq 0$$

- (2) 8. A marketing survey lists 10 types of candy and ask those surveyed to list their favourite, second favourite and least favourite. How many ways can the survey be filled out?
- (4) 9. A survey of 100 rivers shows that 45 are polluted with chemicals, 35 are polluted with pesticides and 20 are polluted with unrefined ore, 12 are polluted with chemicals and pesticides, 10 with chemicals and ore, 8 with pesticides and ore and 5 are polluted with all 3 agents. How many rivers are polluted by
- (a) none of the 3 agents?
 - (b) chemicals and pesticides, but not ore?
 - (c) exactly one of the 3 agents?
- (2) 10. If a fair coin is tossed 4 times, what is the probability that an equal number of heads and tails are obtained?
- (5) 11. Alice, Burt, Candace, David, Ellen and Fred are candidates for 4 jobs. If the jobs are assigned at random,
- (a) What is the probability that Alice, Burt and Candace all get jobs?
 - (b) What is the probability exactly 2 of Alice, Burt and Candace get jobs?
 - (c) What is the probability at least 2 of Alice, Burt and Candace get jobs?
- (3) 12. Of the employees in a corporation, 5% of the men and 7% of the women have salaries above \$50,000. Furthermore, 60% of the employees are men. If an employee who is selected at random earns more than \$50,000, what is the probability that the employee is a man?
- (2) 13. A student borrows \$8,000 for 3 years and the bank charges 6% per annum compounded semi-annually. How much is owed at the end of the 3 years if no intermediate payments are made?

- (3) 14. You want to invest \$10,000 in a GIC for 6 years. Determine which is the better interest rate:
- (a) 8.5% per annum simple interest; or
 - (b) 8% per annum compounded quarterly?
- (3) 15. A family makes monthly deposits into an RESP fund which pays 10% per annum compounded monthly. In order to achieve \$20,000 at the end of 8 years, how much should be deposited at the end of each month?
- (3) 16. A student pays \$500 down and \$200 per month over 5 years for a car. If the finance rate is 12% compounded monthly, what is the selling price of the car?
- (4) 17. A house is bought for \$180,000. There is a down payment of \$30,000. The balance is amortized with monthly payments over 20 years at 6% interest compounded monthly. If the owner wants to pay off the mortgage after 5 years, how much will be owed to the bank?