DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1833

FINAL EXAMINATION — TIME: $2\frac{1}{2}$ HOURS
APRIL 2000 — PROF. D. SULLIVAN — TOTAL MARKS: 70

- Attempt all problems. The examination contains 10 problems in Section A, and 6 problems in Section B.
- Credit will be given for presentation and methods of solution.
- Show all your work. Marks will not be given for "solutions" that contain only the answer to the problem attempted.
- Calculators may be used for calculation purposes only.
- The use of notes, cheat sheets, etc. is not allowed.

SECTION A

MARKS

(2) 1. Determine the equation of the line that passes through $A(-2, 3)$ and is perpendicular to $4x + 5y - 1 = 0$.

(2) 2. Find the coordinates of the point of intersection of the lines, $x - 2y - 3 = 0$ and $-2x + 3y + 4 = 0$.

(2) 3. Evaluate the matrix product, $\begin{bmatrix} 1 & -2 & 3 \\ 5 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & -4 \\ 1 & 2 \end{bmatrix}$.

(2) 4. Determine the inverse of the matrix, $A = \begin{bmatrix} 2 & -1 \\ 3 & -3 \end{bmatrix}$.

(2) 5. For what values of x and y is $\begin{bmatrix} 4 & 3 \\ x & 2 \end{bmatrix}$ the inverse of $\begin{bmatrix} 1 & y \\ -1 & 2 \end{bmatrix}$?

(2) 6. Calculate the future value of \$700 over 5 years under a rate of simple interest of 8% per year.

(2) 7. Calculate the present value of \$1000 over 10 years under a rate of compound interest of 6% per year.

(2) 8. Evaluate $s_{\overline{n}|i}$ for $n = 15$ and $i = 10\%$.

(2) 9. Determine the number of 4-person committees that may be formed from 6 men and 3 women, given that each committee must include at least 2 men.

(2) 10. E_1 and E_2 are two events of a sample space $\mathcal{S}$ for which $P[E_1] = 0.53$, $P[E_2] = 0.41$ and $P[E_1 \cap E_2] = 0.39$. Calculate the probabilities, $P[E_1 \cup E_2]$ and $P[E_1|E_2]$.

SECTION B

(8) 1. Use elementary row operations on matrices to find all solutions of the linear system,

$$\begin{array}{rcrcrcrcr} 2x & + & 3y & - & 3z & + & 4w & = & -2 \\ & & y & - & z & + & 2w & = & 2 \\ 2x & + & 3y & - & z & - & w & = & -7 \\ 2x & + & y & - & z & & & = & -6 \end{array} .$$

(10) 2. (i) Calculate the inverse of the matrix, $A = \begin{bmatrix} -1 & 1 & -1 \\ 2 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$.

(ii) Use the answer to part (i) to determine the unique solution of

$$\begin{array}{rcrcrcr} -x_1 & + & x_2 & - & x_3 & = & -6 \\ 2x_1 & - & x_2 & + & 2x_3 & = & 12 \\ 2x_1 & - & x_2 & + & x_3 & = & 7 \end{array} .$$

(8) 3. Determine the minimum and maximum values of the function, $f(x, y) = -3x + 4y$ over the feasible set described by the inequalities,

$$y \geq 2, \quad y + x \leq 6, \quad y - 2x \leq 0, \quad -y + x \leq 0.$$

(9) 4. (i) Find the future value of \$2500 in 10 years under a rate of interest of 8% compounded every 6 months.

(ii) Calculate the present value of \$30,000 payable in 5 years under a rate of interest of 10% compounded every 3 months.

(iii) Determine the rate of interest, effective per year, that is equivalent to an annual rate of 12% compounded every 4 months.

(8) 5. A loan of \$20,000 is being repaid over 3 years by equal payments at the end of every month, at a rate of interest of 16% compounded monthly. Calculate

(i) the amount of each monthly payment, and

(ii) the amount of interest in the 20th payment.

(7) 6. In the manufacture of alkaline batteries by a certain company, it is known that 20% of all batteries produced are defective. After production, the company ships all the batteries to two retail outlets, R_1 and R_2, where the batteries are packed in packages of six and offered for sale.

(i) Retail company, R_1, does not check any batteries before packaging, but prices each package at the relatively low price of \$3.50. Calculate the probability that a package of batteries bought from this company contains two defective batteries.

(ii) Retail company, R_2, checks all batteries before packaging, discards all defective ones, and prices each package at the higher price of \$4.50. Calculate the probability that an employee in the packing department of R_2 has the check 10 batteries before obtaining six that are not defective, and can be packaged.