

STUDENT'S NAME: _____ ID #: _____

DEPARTMENT OF MATHEMATICS & STATISTICS
UNIVERSITY OF NEW BRUNSWICK
MATH 1833 TEST 3 (DECEMBER 11, 2006)

TIME: 2 HOURS

CALCULATORS ALLOWED, BUT SHOW ALL YOUR WORK

1. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 & 1 \\ 2 & 2 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$.

(a) (2 marks) Compute $A + B$ and $3A - 2B$

(b) (2 marks) Compute $A \cdot C$ and $C \cdot A$

2. (4 marks) Show that the system has infinitely many solutions, and give numerical examples of two different solutions:

$$\begin{aligned}x + y + z &= 1 \\y + z &= 2 \\x + 2y + 2z &= 3\end{aligned}$$

3. Consider the system of equations

$$\begin{aligned}x + 2y &= 2 \\2x + ky &= 10\end{aligned}$$

(a) (2 marks) For what values of k does the system have no solution?

(b) (1 mark) For what values of k does it have exactly one solution?

4. (4 marks) A furniture manufacturer makes chairs, coffee-tables and dining tables. Each piece of furniture requires sanding, staining and varnishing. The time, in minutes, required for these operations on each piece of furniture is given in the following table:

	CHAIR	COFFEE-TABLE	DINING-TABLE
SANDING	2	5	5
VARNISHING	4	3	6
STAINING	2	4	4

The sanding and varnishing stations are each available 360 minutes per day, and the staining station is available 300 minutes per day. How many of each type of furniture can the company produce daily?

5. (3 marks) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

You are told that $A^{-1} = \begin{bmatrix} -1 & 1 & -1 \\ 2 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$. Use A^{-1} to solve the equations $AX = B$ and $AX = C$.

6. (4 marks) Compute the inverse of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$$

7. In the following Linear Programming Problem, x is the amount invested in Project 1, y is the amount invested in Project 2, and x and y are subject to the following constraints:

$$\begin{aligned} x + y &\leq 25 \\ y &\geq x/2 \\ x &\geq 10 \end{aligned}$$

- (a) (3 marks) Determine the solution set (feasible region). Show it in a graph.
- (b) (3 marks) Maximize the function $P = 0.12x + 0.09y$, subject to the constraints given above.
8. (2 marks) Formulate the following situation as a Linear Programming Problem, but **DO NOT SOLVE** the linear programming problem (explain what variables x and y denote, write the inequalities for the feasible region, write the objective function and state whether it should be minimized or maximized):

Jimbo's Charter & Cargo has up to 160 million dollars to purchase aeroplanes for its fleet. The company has two models to choose from: model A and model B . Each model A plane has 400 cubic feet of cargo space and 80 seats. Each model B has 300 cubic feet of cargo space and 100 seats. Jimbo's business analysts determine that its fleet must have a total of at least 48000 cubic feet of cargo space and at least 12800 seats. If each model A and each model B aeroplane costs one million dollars, how many of each model should the company get in order to minimize the cost?