

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 1833

FINAL EXAMINATION  
DECEMBER 1999

TIME: 3 HOURS  
TOTAL POINTS = 60

INSTRUCTIONS:

- (a) Show **ALL** intermediate calculations.
- (b) Calculators are permitted.

FORMULAS:

$$S = P(1 + rt)$$

$$S = P(1 + i)^n$$

$$S = Rs_n|i = R \left[ \frac{(1 + i)^n - 1}{i} \right]$$

$$P = Ra_n|i = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$$

- (3) 1. A market research firm has established the following equations for the Christmas time supply and demand for Silly Elmo hats:

$$\begin{array}{ll} \text{supply} & q = 2p - 12 \\ \text{demand} & q = -3p + 28 \end{array}$$

where  $p$  is the price in dollars per hat and  $q$  is the number (in thousands) of hats to be supplied and bought. Find the equilibrium price and determine the number of hats that will be sold at that price.

- (5) 2. Using the Gauss-Jordan elimination method, solve

$$\begin{array}{rcl} 3x + 2y - 20z & = & 15 \\ x - y + 5z & = & 0 \\ 4x + y - 15z & = & 15 \end{array} .$$

- (5) 3. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} .$$

- (2) (b) Use the answer to part (a) to solve the system of equations

$$\begin{array}{rcl} x + y & + & w = 2 \\ & y + z + 2w & = 0 \\ x - y + z - w & = & 4 \\ & z + w & = 3 \end{array} .$$

(4) 4. Let  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & -2 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 0 & -2 \end{bmatrix}$ ,  $C = \begin{bmatrix} 4 & 5 \\ -6 & 1 \end{bmatrix}$ .

Find, if possible,  $BA - 2C$ .

- (4) 5. A door manufacturer produces doors in two styles, regular and ornate. It costs \$500 to make each regular door, which the manufacturer sells for \$650. It costs \$700 to make each ornate door and each sells for \$975. The daily production capacity is 100 doors and the daily cost cannot exceed \$60,000. How many doors of each type should be made per day to maximize profit? Express this as a linear programming problem. **DO NOT SOLVE IT.**

- (5) 6. Maximize  $P(x, y) = 2x + 3y$   
subject to

$$\begin{aligned} x + 3y &\leq 6 \\ x + 4y &\geq 4 \\ y &\leq 1 \\ x &\geq 0 \\ y &\geq 0 \end{aligned} .$$

- (3) 7. 100 families were surveyed about what pets they had.

60 families had a cat  
40 families had a dog  
15 families had a hamster  
30 families had both a cat and a dog  
5 families had a cat and a hamster  
10 families had a dog and a hamster  
2 families had a cat, a dog and a hamster.

- (a) How many families had only a cat?  
(b) How many families had none of these pets?

8. If  $P(A) = 0.7$ ,  $P(B) = 0.4$ ,  $P(A \cap B) = 0.3$ , find

- (1) (a)  $P(A \cup B)$   
(2) (b)  $P(A' \cap B')$

- (2) 9. Two dice are rolled. What is the probability of getting a total of 11 or 12?

- (2) 10. How many different types of homes are available if a builder offers a choice of 5 basic plans, 3 roof styles and 2 exterior finishes?

- (3) 11. A shipment of 100 jackets contains 12 with faulty zippers. Of all possible samples of size 4, how many contain exactly three jackets with faulty zippers?

- (5) 12. Statistics collected show that the probability that a heavy smoker will die of lung cancer is 30%. For a moderate smoker the probability of dying of lung cancer is 20% while for a non-smoker it is 5%. If 30% of the population are heavy smokers, 50% are moderate smokers and 20% are non-smokers, what is the probability that a person who died of lung cancer was a heavy smoker?

- (3) 13. How much should a 60 year old have invested in RRSP's at 6% compounded semi-annually to have \$120,000 at age 65?
- (2) 14. What is the effective annual interest rate if the stated (nominal) interest rate is 6% compounded monthly?
15. John and Mary take a \$120,000 mortgage to be amortized by monthly payments over 25 years at 6% compounded monthly.
- (3) (a) What is the size of their monthly payment?
- (3) (b) How much will they still owe on their house (remaining loan balance) after 15 years?
- (3) 16. John gives up smoking and gambling. This allows him to save and deposit \$100 at the end of each month in an account yielding 9% compounded monthly. How much will he have in the account after 5 years?