

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2003

FINAL EXAMINATION

December, 2000

Time: 3 Hours

NO CALCULATORS

MARKS

(15) 1. Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, calculate the following:

- (a) $|2\mathbf{a} - 3\mathbf{b}|$,
- (b) the angle between the vectors \mathbf{a} and \mathbf{b} ,
- (c) the area of the triangle with \mathbf{a} and \mathbf{b} as adjacent sides,
- (d) the value of k such that $\mathbf{a} + k\mathbf{b}$ is perpendicular to \mathbf{b} .

(15) 2. (a) Find an equation of the plane through the point $(2, -1, 3)$ and parallel to the plane $3x - y + z = 5$.

(b) Find an equation of the plane passing through the point $p(-2, 4, 5)$ and containing line

$$x = t, \quad y = -2 - 2t, \quad z = 1 + 3t.$$

(c) Find the point where the line

$$x = 2t + 1, \quad y = t - 1, \quad z = -t + 2$$

intersects the plane $x + y - z = 6$.

(10) 3. (a) Suppose that the equation

$$e^{xyz} + x^2yz + z = 2$$

defines z implicitly as a function of x and y , find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$.

(b) Show that $u(x, t) = \sin(x - ct)$ satisfies

$$c^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} = 0,$$

where c is a constant.

(16) 4. (a) Find all local maxima, local minima and saddle points of

$$f(x, y) = x^2 - 2xy + y^3 - y.$$

(b) Use Lagrange multipliers to find the point on the plane

$$5x + y - 2z = 6$$

closest to the origin.

- (10) 5. Let $F(x, y, z) = x^2y + y^2z + z^2x$.
- (a) Find the directional derivative of F at the point $P(2, -1, 1)$ in the direction of $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. In what direction does F change most rapidly at P ?
- (b) Find an equation of the tangent plane to the surface $F(x, y, z) = -1$ at $P(2, -1, 1)$.
- (6) 6. Find the point on the paraboloid $z = \frac{x^2}{9} + \frac{y^2}{18}$ where the tangent plane is parallel to the plane $2x - y + 3z = 1$.

- (12) 7. (a) Use polar coordinates to evaluate

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} e^{x^2+y^2} dy dx.$$

- (b) Sketch the region of integration, reverse the order of integration and evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 x \cos y dx dy.$$

- (18) 8. (a) Use cylindrical coordinates to evaluate

$$\iiint_R x dV$$

where R is the region in the first octant bounded by the paraboloid $z = 4 - x^2 - y^2$ and the coordinate planes.

- (b) Use spherical coordinates to evaluate

$$\iiint_R \sqrt{x^2 + y^2 + z^2} dV$$

where R is the region that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the plane $z = 2$.

- (6) 9. Express the triple integral $\iiint_R f(x, y, z) dV$ as a repeated integral in Cartesian coordinates, where R is the region bounded by the elliptic paraboloid $y = 4x^2 + z^2$ and the plane $y = 9$.

(100)