DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2003

FINAL EXAMINATION

December 1999 Time: 3 Hours

NO CALCULATORS

MARKS

(15) 1. Given three points $A(1,-2,3)$, $B(3,1,2)$ and $C(2,3,-1)$, find:

(a) the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$,

(b) $\cos\theta$, where θ is the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$,

(c) symmetric equations of the line through A and B,

(d) $\overrightarrow{AB} \times \overrightarrow{AC}$,

(e) an equation of the plane through A, B, C.

(10) 2. (a) Find an equation of the plane that passes through the point $p(1,-1,2)$ and is parallel to the plane

$$2x - y + z + 1 = 0.$$

(b) i. Find parametric equations of the line through the point $p(1,-1,2)$ and perpendicular to the plane

$$2x - y + z + 1 = 0.$$

ii. Determine the point of intersection of the line with the plane.

(10) 3. (a) Using the chain rule, find $\dfrac{\partial z}{\partial r}$ if

$$z = e^{x^2 y}, \quad x = r\cos s, \quad y = s\sin r.$$

(b) If $z = f(x,y)$, where $x = r^2 - s^2$, and $y = s^2 - r^2$, prove that

$$s\frac{\partial z}{\partial r} + r\frac{\partial z}{\partial s} = 0.$$

(16) 4. (a) Find all local maxima, local minima and saddle points of

$$f(x,y) = x^2 + 2y^2 - x^2 y.$$

(b) Use Lagrange multipliers to find the maximum and minimum value of

$$f(x,y,z) = x - 2y + 3z$$

subject to the constraint

$$x^2 + 2y^2 + z^2 = 12.$$

(13) 5. Let

$$f(x, y, z) = x^2 + xz + 2y^2 + z.$$

Find:

(a) the directional derivative of f at $(1, -2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$,

(b) the maximum value of the directional derivative of f at $(1, -2, 3)$,

(c) an equation of the tangent plane to the surface $f(x, y, z) = 15$ at $(1, -2, 3)$,

(d) all points on surface $f(x, y, z) = 5$ where the tangent plane is parallel to the plane $y = 0$.

(12) 6. (a) Use polar coordinates to evaluate

$$\iint\limits_R y \, dA$$

where R is the region in the first quadrant bounded by the x-axis and the circle $x^2 + y^2 = 2x$.

(b) Sketch the region of integration, reverse the order of integration and evaluate the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{1 + y^3} \, dy \, dx.$$

(18) 7. (a) Use cylindrical coordinates to evaluate

$$\iiint\limits_R \sqrt{x^2 + y^2} \, dV$$

where R is the region below the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane.

(b) Use spherical coordinates to evaluate

$$\iiint\limits_R (x^2 + y^2 + z^2)^{1/2} \, dV$$

where R is the region inside the sphere $x^2 + y^2 + z^2 = 2z$.

(6) 8. Express the triple integral $\displaystyle\iiint\limits_R f(x, y, z) \, dV$ as a repeated integral in Cartesian coordinates, where R is the region in the first octant bounded by the coordinate planes and the ellipsoid $\dfrac{x^2}{4} + y^2 + z^2 = 1$.

(100)