

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2003

FINAL EXAMINATION

December 1999

Time: 3 Hours

NO CALCULATORS

MARKS

(15) 1. Given three points $A(1, -2, 3)$, $B(3, 1, 2)$ and $C(2, 3, -1)$, find:

- (a) the vectors \vec{AB} and \vec{AC} ,
- (b) $\cos \theta$, where θ is the angle between \vec{AB} and \vec{AC} ,
- (c) symmetric equations of the line through A and B ,
- (d) $\vec{AB} \times \vec{AC}$,
- (e) an equation of the plane through A, B, C .

(10) 2. (a) Find an equation of the plane that passes through the point $p(1, -1, 2)$ and is parallel to the plane

$$2x - y + z + 1 = 0.$$

- (b) i. Find parametric equations of the line through the point $p(1, -1, 2)$ and perpendicular to the plane

$$2x - y + z + 1 = 0.$$

- ii. Determine the point of intersection of the line with the plane.

(10) 3. (a) Using the chain rule, find $\frac{\partial z}{\partial r}$ if

$$z = e^{x^2 y}, \quad x = r \cos s, \quad y = s \sin r.$$

- (b) If $z = f(x, y)$, where $x = r^2 - s^2$, and $y = s^2 - r^2$, prove that

$$s \frac{\partial z}{\partial r} + r \frac{\partial z}{\partial s} = 0.$$

(16) 4. (a) Find all local maxima, local minima and saddle points of

$$f(x, y) = x^2 + 2y^2 - x^2 y.$$

- (b) Use Lagrange multipliers to find the maximum and minimum value of

$$f(x, y, z) = x - 2y + 3z$$

subject to the constraint

$$x^2 + 2y^2 + z^2 = 12.$$

(13) 5. Let

$$f(x, y, z) = x^2 + xz + 2y^2 + z.$$

Find:

- (a) the directional derivative of f at $(1, -2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$,
- (b) the maximum value of the directional derivative of f at $(1, -2, 3)$,
- (c) an equation of the tangent plane to the surface $f(x, y, z) = 15$ at $(1, -2, 3)$,
- (d) all points on surface $f(x, y, z) = 5$ where the tangent plane is parallel to the plane $y = 0$.

(12) 6. (a) Use polar coordinates to evaluate

$$\iint_R y \, dA$$

where R is the region in the first quadrant bounded by the x -axis and the circle $x^2 + y^2 = 2x$.

- (b) Sketch the region of integration, reverse the order of integration and evaluate the integral

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{1 + y^3} \, dy \, dx.$$

(18) 7. (a) Use cylindrical coordinates to evaluate

$$\iiint_R \sqrt{x^2 + y^2} \, dV$$

where R is the region below the paraboloid $z = 4 - x^2 - y^2$ and above the xy -plane.

- (b) Use spherical coordinates to evaluate

$$\iiint_R (x^2 + y^2 + z^2)^{1/2} \, dV$$

where R is the region inside the sphere $x^2 + y^2 + z^2 = 2z$.

(6) 8. Express the triple integral $\iiint_R f(x, y, z) \, dV$ as a repeated integral in Cartesian coordinates, where R is the region in the first octant bounded by the coordinate planes and the ellipsoid $\frac{x^2}{4} + y^2 + z^2 = 1$.

(100)