DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2213

FINAL EXAMINATION
APRIL 2000 TIME: 3 Hours

CALCULATORS ARE NOT PERMITTED. SHOW ALL WORK; CREDIT WILL BE GIVEN FOR PRESENTATION AND METHOD OF SOLUTIONS.

VALUE

(10) 1. Use an LU-decomposition of the coefficient matrix, $A = LU$, to find all solutions of the system of linear equations

$$\begin{array}{rcrcrcrcr} -x_1 & + & 2x_2 & + & x_3 & - & x_4 & = & 1 \\ x_1 & - & 4x_2 & & & + & 5x_4 & = & -2 \\ -2x_1 & + & 6x_2 & - & x_3 & - & 5x_4 & = & -4 \\ -x_1 & - & 4x_2 & + & 4x_3 & + & 11x_4 & = & -2 \end{array}.$$

(10) 2. (i) Compute the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & -4 \\ -1 & 1 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

(ii) Use the result of part (i) to find the inverses of the three matrices A^t, $3A$ and A^2.

(10) 3. Let $A = \begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ -1 & -2 & 1 & 2 & 3 \\ 2 & 4 & -3 & 2 & 0 \\ -3 & -6 & 2 & 0 & 3 \end{bmatrix}$.

(i) Determine a basis for the column space of A.

(ii) If $T : \mathbb{R}^5 \to \mathbb{R}^4$ is the linear transformation induced by A, describe the nullspace of T in terms of a span in $\mathbb{R}^5$.

(10) 4. (a) Let S denote the subset of all vectors $\vec{x}$ in $\mathbb{R}^2$ for which $\vec{x} \cdot \vec{u} = 0$ for some fixed $\vec{u} \in \mathbb{R}^2$. Is S a subspace of $\mathbb{R}^2$ or not? Give reasons for your answer.

(b) Let $\vec{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ be two vectors of $\mathbb{R}^2$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation for which $T(\vec{u}) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $T(\vec{v}) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(i) Find $T(\vec{e}_1)$ and $T(\vec{e}_2)$.

(ii) Find the matrix representation of T with respect to the standard basis $\{\vec{e}_1, \vec{e}_2\}$.

(iii) For what vector $\vec{w}$ of $\mathbb{R}^2$ is $T(\vec{w}) = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$?

(10) 5. (a) Find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

(b) Is A diagonalizable? Give a reason for your answer.

(10) 6. A certain 3×3 matrix A has a single eigenvalue $= 1$ with eigenspace $= \text{span}(2, 1, -2\sqrt{5})$ and a repreated eigenvalue $= 0$ with eigenspace $= \text{span}\{(-1, 2, 0),\ (1, 0, 1/\sqrt{5})\}$.

(a) Find an orthogonal matrix S which diagonalizes A.

(b) Find A.

(6) 7. Find the least squares straight line fit to the points $(0, 1)$, $(2, 0)$, $(3, 1)$ and $(3, 2)$.

(4) 8. Prove: If there is an orthogonal matrix S which diagonalizes the matrix A, then A is symmetric.

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