

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2503

FINAL EXAMINATION

APRIL 2000

TIME: 3 HOURS

NO CALCULATORS.
SHOW ALL INTERMEDIATE CALCULATIONS.

MARKS

- (4) 1. Solve the initial value problem

$$\left(\frac{x^2 + 2x}{y}\right) \frac{dy}{dx} = x + 1, \quad y(1) = \sqrt{3}.$$

- (6) 2. Find the solution to the differential equation

$$y'' - 3y' = 2x + 1 \quad y(0) = 1, \quad y'(0) = 7/9.$$

- (7) 3. Use the method of variation of parameters to solve

$$y'' - 4y' + 5y = e^{2x} / \sin x.$$

- (6) 4. Test the series for convergence or divergence and name the test used:

$$(i) \sum_{n=0}^{\infty} \frac{\sqrt[3]{4+n^2}}{n^3+5}; \quad (ii) \sum_{n=1}^{\infty} \frac{2^{2n}}{n3^n}.$$

- (3) 5. Determine whether $\sum_{n=0}^{\infty} (-1)^n \left[\frac{3n+4}{e(n+1)} \right]^n$ is absolutely convergent, conditionally convergent or divergent. Name the tests used.

- (4) 6. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{\sqrt{2n+1}}.$$

- (4) 7. (a) Find the Maclaurin series for $f(x) = \cosh x$.

(b) Using the series found in part (a), evaluate $\int \cosh(x^3)dx$ as a power series.

(6) 8. Use a power series to find the solution of the following initial value problem:

$$y'' - 2xy' - 2y = 0 \quad y(0) = 3, \quad y'(0) = 0.$$

(5) 9. Use Gauss-Jordan elimination to find all solutions or show that there are none:

$$\begin{array}{cccccc} x_1 & + & 2x_2 & + & 3x_3 & - & 3x_4 & = & 2 \\ 2x_1 & + & 4x_2 & + & 7x_3 & - & 6x_4 & = & 2 \\ x_1 & + & 2x_2 & + & x_3 & - & x_4 & = & 6 \\ & & & & x_3 & - & x_4 & = & -2 \end{array} .$$

(5) 10. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & -1 \end{bmatrix} .$$

(b) Use the inverse found in part (a) to solve the system

$$\begin{array}{cccc} x & & + & z & = & 3 \\ x & + & y & + & z & = & -2 \\ 2x & + & 3y & - & z & = & 6 \end{array} .$$

(3) 11. For which values of p does the following system have

- (i) no solutions?
- (ii) infinitely many solutions?
- (iii) exactly one solution?

$$\begin{array}{ccc} x & + & py & = & 1 \\ px & + & y & = & 1 \end{array}$$

(7) 12. Find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} .$$