DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2503

FINAL EXAMINATION
APRIL 2003 | TIME: 3 HOURS

NO CALCULATORS PERMITTED.

MARKS

1. Solve the differential equations:

(4) (a) $x + 2y\sqrt{x^2+1}\,\dfrac{dy}{dx} = 0, \quad y(0) = 1.$

(4) (b) $\dfrac{dy}{dx} - 2xy = 3x^2e^{x^2}, \quad y(0) = 4.$

(8) 2. Use the method of variation of parameter to solve:

$$y'' + 3y' + 2y = \cos(e^x).$$

(10) 3. A spring with a 2 kg mass attached to it has a natural length of 0.5 m. A force of 2 N will stretch the spring to 0.7 m. It has a damping constant of 8. An external force of $F(t) = 16\cos t$ is applied to the mass. At time zero, the mass is passing through the equilibrium position in the positive direction with a velocity of 1 m/sec.

(a) Set up the differential equation for $x(t)$, the displacement of the mass from the equilibrium position at time t. Include a statement of the initial conditions.

(b) Solve the equation in part (a), to find the position and velocity of the mass at time t.

(3) 4. If $A = \begin{bmatrix} 1 & -1 & 6 \\ 1 & -2 & -1 \\ 3 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 3 \\ 2 & -4 \\ -3 & 1 \end{bmatrix}$, find AB and BA, where possible.

(6) 5. Use Gauss-Jordan elimination to solve the system

$$\begin{array}{rcrcrcrcrcr} 2x_1 & - & 4x_2 & + & 10x_3 & + & 3x_4 & - & 8x_5 & = & 4 \\ x_1 & + & 2x_2 & + & x_3 & - & 2x_4 & & & = & 7 \\ 3x_1 & + & 5x_2 & + & 4x_3 & - & 10x_4 & - & x_5 & = & 10 \end{array}.$$

(6) 6. Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -1 \\ 2 & -1 & 2 \end{bmatrix}.$$

(3) 7. If $A = \begin{bmatrix} t^3 & 0 & t^2 \\ 17 & 0 & 1 \\ t^4 & 2 & -17 \end{bmatrix}$, for what values of t does A^{-1} not exist?

(10) 8. Determine the eigenvalues and eigenvectors for

$$A = \begin{bmatrix} 1 & 8 & -2 \\ 1 & 0 & 0 \\ 0 & 4 & -1 \end{bmatrix}.$$

(3) 9. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3+2^n}{2^{n+2}}$, if it exists.

(6) 10. Do any <u>two</u> of the following. Determine if the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2+1}{n^3+2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!+1}{(n+1)!}$.

(4) 11. Determine if the following series converges absolutely, converges conditionally or diverges:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}.$$

(6) 12. Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x+2}{2}\right)^n.$$

(3) 13. Find the Taylor series for $f(x) = \dfrac{1}{2x+1}$ about $x = -2$.

(3) 14. Find the Maclaurin series for $\displaystyle\int_0^x \frac{e^t - 1}{t}\, dt$.

(8) 15. Use a power series expansion about $x = 0$ to solve the differential equation

$$(1-x^2)y'' - 3xy' - y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(3) 16. Use the Binomial Theorem to find the Maclaurin series for $\sqrt{1-x}$.

---

(90)