

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2503

FINAL EXAMINATION
DECEMBER 1999

TIME: 3 HOURS
TOTAL POINTS = 90

MARKS

- (5) 1. Use Gauss-Jordan elimination to solve:

$$\begin{array}{rcrcrcrcr} 2x & - & y & + & z & = & -1 \\ & x & - & 2y & + & 3z & = & -6 \\ & 3x & + & y & - & z & = & 6 \end{array}$$

- (5) 2. (a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & -3 & -3 \\ 1 & -2 & -2 \\ 2 & -4 & -5 \end{bmatrix}.$$

- (2) (b) Use the answer to part (a) to solve

$$\begin{array}{rcrcrcrcr} x & - & 3y & - & 3z & = & 3 \\ x & - & 2y & - & 2z & = & 1 \\ 2x & - & 4y & - & 5z & = & 2 \end{array}.$$

- (5) 3. Find the determinant of

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 4 & -2 & -3 \end{bmatrix}.$$

4. Given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 3 \\ 1 & -2 & -1 \end{bmatrix}$,

- (6) (a) find its eigenvalues.
(5) (b) Determine all the eigenvectors corresponding to the largest eigenvalue of A .
5. Test each series for convergence. If it is an alternating series, determine if it is absolutely or conditionally convergent or divergent.

(5) (a) $\sum_{n=1}^{\infty} \frac{n^2 + 1}{3n^2 + 2n - 1}$

(5) (b) $\sum_{n=1}^{\infty} \left[\frac{3}{n^{4/3}} + \left(\frac{3}{4}\right)^n \right]$

(5) (c) $\sum_{n=2}^{\infty} ne^{-n^2/2}$

- (5) 6. (a) Find the Taylor series for $f(x) = \sin(2x)$ expanded around the centre $a = \pi/4$.
(7) (b) Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{n}{2n^2 + 3} \left(\frac{x+1}{3} \right)^n.$$

7. Consider the linear differential equation of order 2:

$$y'' + xy' = 0.$$

- (5) (a) Substitute the power series

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

into the differential equation. Find the recursion relation for the coefficients a_n .

- (4) (b) Write the first four non-zero terms in the power series solution.
(3) (c) Find the general n^{th} coefficient a_n and use this to write the explicit power series solution in Σ notation.

- (7) 8. Solve the initial value problem

$$\frac{dy}{dt} + y = (t+1)^2 \quad y(0) = 0.$$

- (8) 9. Solve the differential equation:

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}.$$

- (8) 10. A series circuit consists of a resistor with $R = 20 \Omega$, an inductor with $L = 1 \text{ H}$, a capacitor with $C = 0.002 \text{ F}$, and a 12-V battery. If the initial charge and current are both 0, find the charge and current at time t .