DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2503

FINAL EXAMINATION — TIME: 3 HOURS
DECEMBER 1999 — TOTAL POINTS = 90

MARKS

(5) 1. Use Gauss-Jordan elimination to solve:

$$\begin{array}{rcrcrcr} 2x & - & y & + & z & = & -1 \\ x & - & 2y & + & 3z & = & -6 \\ 3x & + & y & - & z & = & 6 \end{array}$$

(5) 2. (a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & -3 & -3 \\ 1 & -2 & -2 \\ 2 & -4 & -5 \end{bmatrix}.$$

(2) (b) Use the answer to part (a) to solve

$$\begin{array}{rcrcrcr} x & - & 3y & - & 3z & = & 3 \\ x & - & 2y & - & 2z & = & 1 \\ 2x & - & 4y & - & 5z & = & 2 \end{array}.$$

(5) 3. Find the determinant of

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 3 & 2 & 1 \\ 2 & 4 & -2 & -3 \end{bmatrix}.$$

4. Given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 3 \\ 1 & -2 & -1 \end{bmatrix}$,

(6) (a) find its eigenvalues.

(5) (b) Determine all the eigenvectors corresponding to the largest eigenvalue of A.

5. Test each series for convergence. If it is an alternating series, determine if it is absolutely or conditionally convergent or divergent.

(5) (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{3n^2+2n-1}$

(5) (b) $\displaystyle\sum_{n=1}^{\infty} \left[\frac{3}{n^{4/3}} + \left(\frac{3}{4}\right)^n\right]$

(5) (c) $\displaystyle\sum_{n=2}^{\infty} ne^{-n^2/2}$

(5) 6. (a) Find the Taylor series for $f(x) = \sin(2x)$ expanded around the centre $a = \pi/4$.

(7) (b) Find the <u>interval</u> of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{n}{2n^2+3} \left(\frac{x+1}{3}\right)^n.$$

7. Consider the linear differential equation of order 2:

$$y'' + xy' = 0.$$

(5) (a) Substitute the power series

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

into the differential equation. Find the recursion relation for the coefficients a_n.

(4) (b) Write the first <u>four</u> non-zero terms in the power series solution.

(3) (c) Find the general nth coeffcient a_n and use this to write the explicit power series solution in Σ notation.

(7) 8. Solve the initial value problem

$$\frac{dy}{dt} + y = (t+1)^2 \qquad y(0) = 0.$$

(8) 9. Solve the differential equation:

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}.$$

(8) 10. A series circuit consists of a resistor with $R = 20\ \Omega$, an inductor with $L = 1$ H, a capacitor with $C = 0.002$ F, and a 12-V battery. If the initial charge and current are both 0, find the charge and current at time t.