

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2513

FINAL EXAMINATION

APRIL 2000

TIME: 3 HOURS

**NO CALCULATORS PERMITTED**

MARKS

1. Given  $\vec{a} = \langle 3, -2, 5 \rangle$ ,  $\vec{b} = \langle 2, 2, -2 \rangle$  and  $\vec{c} = \langle 5, 0, 12 \rangle$  calculate the following:

- (2) (a) the cosine of the angle between  $\vec{b}$  and  $\vec{c}$ ;
- (3) (b) the vector projection of  $\vec{a}$  onto  $\vec{c}$ ;
- (4) (c) the volume of the parallelepiped determined by the vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ .
- (5) 2. (a) Find the parametric **and** symmetric equations of the line which is parallel to the line of intersection of the planes  $x + y + z = 1$  and  $2x - 3y + z = 4$ , and which passes through  $P(1, -1, 2)$ .
- (4) (b) Find the equation of the plane which contains the point  $P(1, 0, -1)$  and the line  $L : \frac{x}{1} = \frac{y-1}{2} ; z = 2$ .
- (4) 3. (a) Find the equation of the tangent plane to the surface  $x^2 - y^2 + z^2 = 4$  at the point  $(2, -3, 3)$ .
- (4) (b) Find the directional derivative of

$$F(x, y, z) = \ln(x^2 + y^2) + e^z$$

at the point  $P(0, 1, 0)$  in the direction  $\vec{v} = \langle -4, 1, 3 \rangle$ .

- (4) (c) Use the chain rule to find  $\frac{\partial z}{\partial r}$  if  $z = \sqrt{x^4 + 4xy}$  with  $x = s^2 \cos r$  and  $y = s^4 \tan r$ .
- (7) 4. (a) Classify the critical points of

$$f(x, y) = x^3 + y^2 - 6xy + 6x + 3y - 7.$$

- (6) (b) Use Lagrange multipliers to find the minimum value of  $f(x, y, z) = x^2 + y^2 + z^2$  subject to the constraint  $3x + 4y + 12z = 13$ .

- (5) 5. (a) Sketch the region of integration, change the order of integration and evaluate

$$\int_0^1 \int_{x^2}^1 x^3 \sqrt{1 + y^3} dy dx.$$

- (7) (b) Use polar coordinates to evaluate  $\int \int_R y dA$  where  $R$  is the region in the 1st quadrant which is inside the circle  $x^2 + y^2 = 2x$ .

- (5) 6. (a) Use cylindrical coordinates to find the volume of the solid bounded by the paraboloids  $z = x^2 + y^2$  and  $z = 36 - 3x^2 - 3y^2$ .
- (5) (b) Use spherical coordinates to evaluate

$$\int \int \int_R (x^2 + y^2 + z^2)^{1/2} dV$$

where  $R$  is the region bounded above by the sphere  $x^2 + y^2 + z^2 = 4$  and below by the cone  $x^2 + y^2 = z^2$ .

- (4) 7. (a) Evaluate  $\int_C (xydx - ydy + dz)$  along the arc  $C$  given by  $x = t, y = t^2, z = t^3, 0 \leq t \leq 1$ .
- (6) (b) Show that

$$\vec{F}(x, y) = \langle 2xy + \cos(x + y), x^2 + y + \cos(x + y) \rangle$$

is conservative and find a function  $\phi$  such that  $\vec{F} = \vec{\nabla}\phi$ . Use  $\phi$  to evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the arc of an ellipse going from  $(0, 0)$  to  $(2, 1)$ .

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