DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2513

FINAL EXAMINATION
APRIL 2000 TIME: 3 HOURS

NO CALCULATORS PERMITTED

MARKS

1. Given $\vec{a} = \langle 3, -2, 5\rangle$, $\vec{b} = \langle 2, 2, -2\rangle$ and $\vec{c} = \langle 5, 0, 12\rangle$ calculate the following:

(2) (a) the cosine of the angle between $\vec{b}$ and $\vec{c}$;

(3) (b) the vector projection of $\vec{a}$ onto $\vec{c}$;

(4) (c) the volume of the parallelepiped determined by the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(5) 2. (a) Find the parametric **and** symmetric equations of the line which is parallel to the line of intersection of the planes $x + y + z = 1$ and $2x - 3y + z = 4$, and which passes through $P(1, -1, 2)$.

(4) (b) Find the equation of the plane which contains the point $P(1, 0, -1)$ and the line $L : \dfrac{x}{1} = \dfrac{y-1}{2}$; $z = 2$.

(4) 3. (a) Find the equation of the tangent plane to the surface $x^2 - y^2 + z^2 = 4$ at the point $(2, -3, 3)$.

(4) (b) Find the directional derivative of

$$F(x, y, z) = \ln(x^2 + y^2) + e^z$$

at the point $P(0, 1, 0)$ in the direction $\vec{v} = \langle -4, 1, 3\rangle$.

(4) (c) Use the chain rule to find $\dfrac{\partial z}{\partial r}$ if $z = \sqrt{x^4 + 4xy}$ with $x = s^2 \cos r$ and $y = s^4 \tan r$.

(7) 4. (a) Classify the critical points of

$$f(x, y) = x^3 + y^2 - 6xy + 6x + 3y - 7.$$

(6) (b) Use Lagrange multipliers to find the minimum value of $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraint $3x + 4y + 12z = 13$.

(5) 5. (a) Sketch the region of integration, change the order of integration and evaluate

$$\int_0^1 \int_{x^2}^1 x^3 \sqrt{1 + y^3}\, dy\, dx.$$

(7) (b) Use polar coordinates to evaluate $\displaystyle\iint_R y\,dA$ where R is the region in the lst quadrant which is inside the circle $x^2 + y^2 = 2x$.

(5) 6. (a) Use cylindrical coordinates to find the volume of the solid bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

(5) (b) Use spherical coordinates to evaluate

$$\iiint_R (x^2 + y^2 + z^2)^{1/2} dV$$

where R is the region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $x^2 + y^2 = z^2$.

(4) 7. (a) Evaluate $\int_C (xy dx - y dy + dz)$ along the arc C given by $x = t,\ y = t^2,\ z = t^3,\ \ 0 \le t \le 1$.

(6) (b) Show that

$$\vec{F}(x, y) = \langle 2xy + \cos(x + y),\ x^2 + y + \cos(x + y)\rangle$$

is conservative and find a function ϕ such that $\vec{F} = \vec{\nabla}\phi$. Use ϕ to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the arc of an ellipse going from $(0, 0)$ to $(2, 1)$.

---

(75)