DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2513

FINAL EXAMINATION
DECEMBER 2000 — TIME: 3 HOURS

NO CALCULATORS PERMITTED

MARKS

1. Given $\vec{a} = (1, -1, 1)$, $\vec{b} = (2, 3, 0)$ and $\vec{c} = (5, 0, 12)$ calculate the following:

(3) (a) the cosine of the angle between $\vec{b}$ and $\vec{c}$;

(3) (b) the vector projection of $\vec{a}$ onto $\vec{c}$;

(3) (c) $\vec{c} \cdot (\vec{a} \times \vec{b})$

(4) 2. (a) Find the parametric **and** symmetric equations of the line which is parallel to the line of intersection of the planes $x + y = 2$ and $-2x + z = 0$, and which passes through the origin.

(3) (b) Find the equation of the plane which contains the point $(1, 0, 0)$ and the line $L : \dfrac{x-1}{2} = y + 1 = \dfrac{z}{3}$.

(4) 3. (a) Find the equation of the tangent plane to the surface $x^2 + 4y^2 + 9z^2 = 25$ at the point $(0, 2, -1)$.

(4) (b) Find the directional derivative of

$$F(x, y, z) = \sin(xyz)$$

at the point $(0, 1, -1)$ in the direction $\vec{v} = (1, -1, 2)$.

(4) (c) Use the chain rule to find $\dfrac{\partial z}{\partial r}$ if $z = x^2 - y^2$ with $x = re^t$ and $y = re^{-t}$.

(5) 4. (a) Find and classify the critical points of

$$f(x, y) = x^3 - 6xy + 8y^3.$$

(5) (b) Use Lagrange multipliers to find the maximum and minimum value of $f(x, y, z) = \dfrac{1}{x} + \dfrac{1}{y}$ subject to the constraint $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 1$.

(5) 5. (a) Sketch the region R of integration bounded by $y^2 = x^3$ and $y = x$, and compute $\displaystyle\int\int_R xy\, dA$.

(5) (b) Use polar coordinates to evaluate $\displaystyle\int\int_R x dA$ where R is the region in the 2nd quadrant which is inside the circle $x^2 + y^2 = 6y$.

(4) 6. Use cylindrical coordinates to find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$.

(4) 7. (a) Evaluate $\int_C x \sin y \, dx + xyz \, dz$ along the arc C given by $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k},\ 0 \leq t \leq 1$.

(4) (b) Show that

$$\vec{F}(x, y) = (2x + y^2 + 3x^2y)\vec{i} + (2xy + x^3 + 3y^2)\vec{j}$$

is conservative and find a function ϕ such that $\vec{F} = \vec{\nabla}\phi$. Use ϕ to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the arc of an ellipse going from $(0, 0)$ to $(2, 1)$.

(60)