

Department of Mathematics & Statistics
MATH 2513

DECEMBER 2005

TIME: 3 HOURS

CALCULATORS NOT PERMITTED

MARKS

- (4) 1. (a) Find an equation of the tangent plane to the surface $x^3 + xy^2 + yz^2 = 1$ at the point $(2, -1, 3)$.
- (4) (b) Find the directional derivative of $f(x, y, z) = \ln(x^2 + 2y^2 + 3z^2)$ at $(2, 1, 1)$ in the direction of $\mathbf{v} = \mathbf{i} - \mathbf{j} - \mathbf{k}$. What is the direction of greatest increase of f at $(2, 1, 1)$?
- (4) 2. (a) Let $f(x, y) = \sqrt{xy}$. Use either differentials or (equivalently) a linear approximation to approximate $f(1.9, 18.2)$.
- (4) (b) Use the chain rule to express $\frac{\partial z}{\partial s}$ as a function of s and t , given that $z = \ln(2x - 3y)$ where $x = se^t$, $y = se^{-t}$.
- (7) 3. (a) Find and classify all local maxima, local minima, and saddle points of

$$f(x, y) = y^3 + x^2 - 6xy + 3x + 6y .$$

- (7) (b) Let $f(x, y, z) = 2x + 6y + 10z$. Use Lagrange Multipliers to find the maximum and minimum values for f subject to the constraint $x^2 + y^2 + z^2 = 35$.

- (4) 4. (a) Use polar coordinates to evaluate

$$\int \int_R \sqrt{x^2 + y^2} \, dA$$

where R is the region bounded by the circle $x^2 + y^2 = 2y$.

- (5) (b) Sketch the region of integration, change the order of integration and evaluate the integral:

$$\int_0^2 \int_{\frac{y}{2}}^1 \sqrt{1+x^2} \, dx \, dy .$$

- (5) 5. Use cylindrical coordinates to evaluate

$$\int \int \int_R \sqrt{x^2 + y^2} \, dV$$

where R is the region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 3$.

- (5) 6. Use spherical coordinates to evaluate

$$\int \int \int_R z \, dV$$

where E is the solid that lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$, in the first octant.

- (6) 7. Evaluate

$$\int_C (-yx^2)dx + xy^2 \, dy \, ,$$

where C is the unit circle $x^2 + y^2 = 1$, oriented counterclockwise, using Green's Theorem.

8. Let $\mathbf{F}(x, y, z) = y\mathbf{i} + (x + z)\mathbf{j} + y\mathbf{k}$.

- (4) (a) Find a potential function f for \mathbf{F} . (That is, find f such that $\nabla f = \mathbf{F}$.)

- (4,2) (b,c) Let C be the line segment from $(2, 0, 4)$ to $(8, 3, 0)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ by two methods: directly and using the fundamental theorem for line integrals (i.e., use a potential function)..

- (4) 9. Compute the curl and divergence of the vector field $\mathbf{F}(x, y, z) = xz\mathbf{i} + xyz\mathbf{j} - y^2\mathbf{k}$.

10. Consider the surface S

$$\mathbf{r}(u, v) = u \cos v \mathbf{i} + u \sin v \mathbf{j} + v \mathbf{k}, \quad 0 \leq u \leq 1, \quad 0 \leq v \leq \pi.$$

- (4) (a) Find an equation of the tangent plane to the surface at $\mathbf{r}(1/2, \pi/2)$.

- (4) (b) Compute the value of the surface integral $\int \int_S \sqrt{1 + x^2 + y^2} \, dS$.