DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2513

FINAL EXAMINATION
APRIL 2005 **TIME: 3 HOURS**

NO CALCULATORS

MARKS

(17) 1. Let S be the surface: $z = f(x, y) = x^2 e^y$.

(a) Find the equations for the tangent plane to S at $P(1, 0, 1)$, and the normal line to S at P.

(b) Evaluate ∇f at the point $(1, 0)$. Compute the directional derivative of $f(x, y)$ at $(1, 0)$ in the direction of the vector $\mathbf{u} = \langle 4, 3\rangle (= 4\mathbf{i} + 3\mathbf{j})$.

(c) Let $x = r\cos\theta,\ y = r\sin\theta$. Use the chain rule to express $\dfrac{\partial z}{\partial r}$ as a function of r and θ.

(d) Let $F(x, y, z) = x^2 e^y - z$. Evaluate ∇F at the point $P(1, 0, 1)$.

(4) 2. Use the total differential to approximate: $(10.08)^2(20.05)$.

(7) 3. Locate, and classify, the critical points of the surface: $f(x, y) = x^3 + xy - y^3$.

(7) 4. Use Lagrange multipliers to find the point on the plane: $5x + y - 2z = 6$ closest to the origin.

(9) 5. (a) Given the double integral $\displaystyle\int_0^1 \int_{\sqrt{x}}^1 \sqrt{1 + y^3}\, dy\, dx$, sketch the region over which integration is performed. Change the order of integration and then evaluate it.

(b) Use a double integral to find the area of the region bounded by $x = 0$, $y = x^2,\ x + y = 2$ in the first quadrant.

(11) 6. (a) Using cylindrical co-ordinates, find the volume of the region inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $(x - 1)^2 + y^2 = 1$.

(b) Using spherical co-ordinates, find the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 9$ and outside the cone $z^2 = x^2 + y^2$.

(6) 7. Show that the force

$$\mathbf{F} = \left\langle x - \frac{1}{1+y^2},\ \frac{2xy}{(1+y^2)^2} \right\rangle = \left(x - \frac{1}{1+y^2} \right) \mathbf{i} + \frac{2xy}{(1+y^2)^2}\,\mathbf{j}$$

is a conservative vector field. Use this fact to find the work it does on a particle as it moves along the upper half of the circle $x^2 + y^2 = 9$ from $(-3, 0)$ to $(3, 0)$.

(4) 8. Use Green's Theorem to find $\oint_C (-y^2\ dx + x\ dy)$ where C is the circle $x^2 + y^2 = 4$ travelled counterclockwise.

(8) 9. (a) Given that

$$\mathbf{F} = \langle z^2 - 2y^2 + 1,\ -4xy,\ \ 2xz - 2 \rangle,$$

find the divergence and curl of $\mathbf{F}$.

(b) Is it possible to express $\mathbf{F}$ of part (a) as the gradient of a function f? If so, find f.

(7) 10. State Stokes' Theorem. Use it to evaluate

$$\oint_C \mathbf{F} \cdot d\mathbf{r}$$

where

$$\mathbf{F} = \langle y, xz, yz \rangle = y\mathbf{i} + xz\mathbf{j} + yz\mathbf{k}$$

and C is the intersection of the plane $x + y + z = 1$ with the three coordinate planes travelled clockwise as viewed from the origin.

(80)