

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 3243

FINAL EXAMINATION

APRIL 2001

TIME: 3 HOURS

VALUE

1. Express in the form $a + bi$, $a, b \in \mathbb{R}$

- (3) (a) $(1 + i)^{17}$;
(3) (b) all solutions of $e^{2iz} = 3$;
(3) (c) all solutions of $\sin z = -3i$.

(6) 2. State the Cauchy-Riemann equations and verify that they are satisfied for the function $f(z) = \sin 2z$.

3. Compute the following integrals:

- (6) (a) $\oint_C \frac{e^z}{z(z-2)^3} dz$ where C is the curve $|z| = 3$ travelled counterclockwise.
(5) (b) $\oint_C \frac{1}{z^2 \sin z} dz$ where C is the curve $|z| = 1$ travelled clockwise.

(5) 4. Classify the singular points of

(a) $\frac{z^3 + 1}{z^2(z + 1)}$ (b) $z^3 e^{1/z}$

(9) 5. Find all Laurent series for

$$f(z) = \frac{z}{(z + 1)(z - 2)}$$

about $z = 0$.

6. Find the principal value of the following integrals:

- (8) (a) $\int_0^\infty \frac{x^2 + 1}{x^4 + 1} dx$
(8) (b) $\int_0^\infty \frac{x^{\lambda-1}}{x - 4} dx$ $0 < \lambda < 1$

(2) 7. Determine the behaviour at ∞ of $f(x) = \frac{\sin z}{z^2}$.

(6) 8. Consider the linear fractional transformation that sends

$$\begin{array}{l} -1 \text{ to } -1 \\ i \text{ to } 0 \\ 1 \text{ to } 1. \end{array}$$

(a) Find the transformation.

(b) What does the interior of $|z| = 1$ get mapped to?

(c) What are the fixed points?

(5) 9. Find a linear fractional transformation that sends the lower half-plane ($\text{Im } z \leq 0$) onto $|z + 1| \leq 1$.

(6) 10. Determine a Schwarz-Christoffel transformation which takes the upper half-plane ($\text{Im } z \geq 0$) onto the strip $-\frac{\pi}{2} \leq \text{Re } w \leq \frac{\pi}{2}$, $\text{Im } w \geq 0$.

(75)