

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 3243

FINAL EXAMINATION

DECEMBER 2003

NO CALCULATORS

TIME: 3 HOURS

MARKS

1. Express in the form $a + bi$, where $a, b \in \mathbb{R}$
- (5) (a) $(-1 - \sqrt{3}i)^{1/4}$;
- (4) (b) $\sin^{-1} 5$;
- (4) (c) $(1 + i)^{\sqrt{3}i}$.
- (5) 2. (a) Show that the Cauchy-Riemann equations imply that if $f(z) = u(x, y) + iv(x, y)$ is analytic then both u and v are harmonic functions.
- (4) (b) Find the harmonic conjugate to $u(x, y) = x^2 - y^2 + x$.
- (6) 3. Find $\oint_C \operatorname{Re}(z) dz$ where $C : |z| = 1$ travelled counterclockwise.
- (6) 4. Find $\oint_C \frac{1}{z^3(z-1)^2} dz$ where $C : |z-2| = 5$ travelled counterclockwise.
- (6) 5. Find all the Laurent series for $f(z) = \frac{1}{z(z-1)}$ about $z = 2$.
- (8) 6. Find *p.v.* $\int_{-\infty}^{\infty} \frac{1}{(x^2+4)(x^2+9)} dx$.
- (8) 7. Find $\int_0^{2\pi} \frac{\cos \theta}{3 + \sin \theta} d\theta$.
- (5) 8. Under the transformation $w = \frac{1}{z}$, find what the line $Ax + By = C$, $C \neq 0$, gets transformed onto.
- (5) 9. Give an analytic expression in x and y for the branch of $\ln z$ which has a branch cut along the positive imaginary axis.
- (9) 10. Show that *p.v.* $\int_0^{\infty} \frac{x^{1/3}}{(x+1)^2} dx = \frac{2\pi}{3\sqrt{3}}$.